The νMSM
and baryogenesis trough sterile neutrino oscillations

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Abstract

We introduce the νMSM, the minimal extension to the standard model of particle physics able to explain the observed neutrino oscillations, dark matter and the baryon asymmetry. First, we discuss in detail the parameters of the theory and the problem of the baryon asymmetry. Then we perform an order of magnitude analysis to prove that the observed baryon asymmetry can be generated and to find the parameter range required for this. As the lightest sterile neutrino, working as dark matter, has mass of the order of 3 keV, the two heavier ones have masses of order GeV and are very degenerate in mass. After this we discuss the possible method of experimentally verifying or, indeed, falsifying the theory.

Contents

Chapter 1

Introduction

The minimal standard model of particle physics has been tremendously successful in describing the particles and forces that make up our world. It uses just 12 fundamental particles of matter, 3 basic interactions, namely the electromagnetic, weak and strong interactions, and the Higgs particle. Our world, everything we can see or touch, consists mostly of just three of the 12 matter particles: the up- and down quarks, forming protons and neutrons, and the electron. There are three generation of different masses of these particles, but the more massive ones decay rapidly into these three particles. All these particles are fundamentally massless and the role of the Higgs particle is to give them mass.

The existence of the neutrino was first suggested by Wolfgang Pauli in 1930. It was needed to explain the fact that some energy was lost in the beta decay, in which a proton emits a positron and becomes a neutron. It had to be very difficult to observe, and therefore had no charge. It would only interact trough the weak interaction, the one responsible for the beta decay. It also needed to be very light. In the standard model it was assumed to be completely massless. This is in agreement with any direct measurements of the neutrino mass [4], but during the last decade the non-zero mass of the neutrino has been established trough other phenomena, the neutrino oscillations. Particles always interact as so called interactions states. However, they propagate as eigenstates of mass, which are linear combinations of the interaction states. Actually, by the word particle we usually mean the mass states. If the mass and interaction states are not the same, particles will oscillate between interaction states while propagating. This oscillation between different types of neutrinos is now believed to explain anomalies measured in the amounts of atmospheric and solar neutrinos [1, 2, 3, 5].

Fermions may have left-handed or right-handed chiral states. This has to do with how it transforms in coordinate transformations. Only the left-handed chiral states interact trough the weak interaction. In the standard model there are only left-handed neutrinos, since right-handed ones would not interact with any other particle. Mass, however, mixes the chiral states, meaning that massive particles however always have both left-handed and right-handed states. If neutrinos have mass, there should be right-handed neutrinos, too.

During the last few decades our knowledge of the Universe has advanced greatly. Observations on the microwave background radiation and the structure of galaxies have made it possible to create a standard model of cosmology. It is not a full theory about the structure of the Universe, but collection of phenomena, explanations and problems that are widely accepted. These include the dark matter, dark energy and the baryon asymmetry. We know that there has to be more mass in galaxies than is visible. The stars at the outskirts of galaxies move faster than they should, if the visible matter was all there is. They behave as if there was a halo of undetectable matter around the galaxy. This is called the dark matter. The Universe is expanding, but according to the Friedmann models derived from the Einstein field-equations, the expansion should be slowing down, due to the effect of gravity. This is not the case, the expansion seems to be speeding up, not slowing down. This can explained by adding a cosmological constant to the field-equations, giving the vacuum a nonzero energy, the dark energy.

In the standard model of particle physics every particle has a counterpart, an antiparticle. The model is almost symmetrical with respect to the exchange of matter an antimatter, and therefore one would expect to find about the same amount of both in the Universe. However we have not been able to find any antimatter in the solarsystem or indeed in particle radiation coming from other systems. The latter suggests that there is much more matter than antimatter even outside the reach of our probes. This asymmetry between matter and antimatter is called the baryon asymmetry of the Universe. As the matter around us consists mostly of baryon, this tells us that there should be a way to violate the baryon number conservation. There actually is a way to violate the baryon number B in the standard model. There is an instanton solution of the classical equations of state of the standard model, which allows the conversion of lepton number L in to baryon number. However $B - L$ should always be conserved.

The neutrino oscillations suggest that neutrinos have mass. This means that we have to add right-handed neutrinos to the standard model. The ν Minimal Standard Model, νMSM, was designed by Mikhail Shaposhnikov and his collaborators to explain both the dark matter and the baryon asymmetry of the Universe adding nothing but the right-handed neutrinos to the standard model [6]. This limits the parameters of the theory greatly, leaving us with a fine-tuning problem: the masses of the right-handed neutrinos have to be selected carefully to get the right phenomenology. The simplicity of the model is however compelling and the small parameter space might make it experimentally testable. Furthermore, a possible way of generating these parameters is suggested in ref. [11].

In this paper we first introduce the structure νMSM. We will explicitly write down the masses of neutrinos and introduce the mixing angles and eigenstates of mass. We will then introduce the problem of baryon asymmetry. Next we will do the calculations of feynman diagrams in finite temperature, needed in the analysis of the asymmetry. Taking into account the constraints given by the fact that we want the lightest of the right-handed, or sterile, neutrinos to work as dark matter, we will do an order of magnitude analysis on the production of baryon asymmetry, proving that there is a range of parameters in which both physical phenomena can be explained. In the last chapter we describe some experiments suggested to either disprove the νMSM completely or to scan its parameter range.

We will assume that the reader has some knowledge of quantum field theory and standard cosmology. Some knowledge of statistical mechanics is also assumed. Field theory in non-zero temperatures will not be introduced in any detail, but some explanation will be provided. Nevertheless our aim is to make this paper as understandable as possible. By standard model we mean the standard model of particle physics, not the cosmological standard model. This includes quantum chronodynamics (strong interactions, QCD) and the electroweak theory with massless neutrinos. In all calculation we will use natural units given by $c = 1$, $\hbar = 1$ and $k_B = 1$ and use the metric convention (+,−,−,−).

Chapter 2

The νMSM

2.1 Neutrino oscillations

In the standard model, neutrinos are assumed to be massless. Adding mass makes oscillations between the three types of neutrinos possible. All particles interact as interaction states and propagate as mass states. They are therefore created and destroyed as interaction states but these states get mixed as they propagate. We may consider the mixing of just two particles. The interaction states ν_e and ν_μ can generally be written as linear combinations of the mass states ν_1 and ν_2 as

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{2.1}$$

After its creation a particle, say ν_e, evolves as its mass eigenstates $\cos(\theta)\nu_1 + \sin(\theta)\nu_2$. If $\hat{H}$ is the hamiltonian operator of the system,

$$\nu_e(t) = e^{-i\hat{H}t}\nu_e = e^{-iE_1 t}\cos(\theta)\nu_1 + e^{-iE_2 t}\sin(\theta)\nu_2$$

We can now write it using the interaction states at some time t:

$$\begin{aligned}\nu_e(t) &= e^{-iE_1 t}\cos(\theta)\left(\cos(\theta)\nu_e - \sin(\theta)\nu_\mu\right) \\ &\quad + e^{-iE_2 t}\sin(\theta)\left(\sin(\theta)\nu_e + \cos(\theta)\nu_\mu\right) \\ &= e^{-iE_1 t}\cos^2(\theta)\nu_e + e^{-iE_2 t}\sin^2(\theta)\nu_e \\ &\quad + (e^{-iE_1 t} + e^{-iE_2 t})\cos(\theta)\sin(\theta)\nu_\mu\end{aligned}$$

As neutrinos are very light particles, we can safely assume that the momentum p is much larger than the masses m_i of the mass states and write

$$E_i = \sqrt{p^2 + m_i^2} \approx p + \frac{m_i^2}{2p},$$

where $i = 1, 2$. This assumption also means that the velocity of the neutrino $v_e \approx 1$, in natural units, which in turn implies that the propagated distance l at time t is $l \approx t$. Taking these into account we get

$$\nu_e(t) = e^{-ipl} e^{-i\frac{m_1^2}{2p}l} \cos^2(\theta)\nu_e + e^{-i\frac{m_2^2}{2p}l} \sin^2(\theta)\nu_e$$
$$+ e^{-ipl}(e^{-i\frac{m_1^2}{2p}l} + e^{-i\frac{m_2^2}{2p}l}) \cos(\theta)\sin(\theta)\nu_\mu.$$

The probability of finding the neutrino in a specific state is the absolute value squared of the coefficient. For example the probability for the neutrino that was created as ν_e to be in the state ν_μ is

$$P(\nu_e \to \nu_\mu) = \sin^2(\theta)\cos^2(\theta)\left(2 + e^{i\frac{m_1^2 - m_2^2}{2p}l} + e^{-i\frac{m_1^2 - m_2^2}{2p}l}\right)$$
$$= \sin^2(2\theta)\sin^2\left(\left|m_1^2 - m_2^2\right| \frac{l}{4p}\right) \tag{2.2}$$

From 2.2 we see that the oscillations depend on the mass squared differences $\Delta m^2 = \left|m_1^2 - m_2^2\right|$, the energy of the neutrino proportional to p and our distance from the source l. Measurement of solar neutrinos, created in the sun in nuclear reactions, give us a mass squared difference of $\Delta m^2_{sol} = (7.66 \pm 0.35)10^{-5}\text{eV}^2$. Atmospheric neutrinos, created as cosmic rays decay into a pion and on to a myon and three neutrinos, give us $\Delta m^2_{atm} = (2.38 \pm 0.27)10^{-3}\text{eV}^2$. [4] The existence of two different mass scales implies that at least two neutrinos have mass. It is not possible to determine the masses absolutely, but the sign of the solar mass difference Δm^2_{sol} can be determined. This leaves us with two possible hierarchies of the masses, the normal hierarchy with $m_1 \leq 15\text{eV}$, $m_2 - m_1 \approx 0.009\text{eV}$ and $m_3 - m_1 \approx 0.050\text{eV}$, and the inverted hierarchy with $m_3 \leq 15\text{eV}$, $m_1 - m_3 \approx 0.048\text{eV}$ and $m_2 - m_3 \approx 0.049\text{eV}$. [5]

2.2 Masses of fermions

In a standard field theory the mass m of a particle appears in its lagrangian as a term $-m\bar{\psi}\psi$. This is usually taken as a part of the free particle lagrangian

$$L = \bar{\psi} i \not{\partial} \psi - m\bar{\psi}\psi \tag{2.3}$$

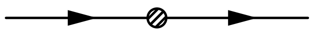
The free particle propagator G is defined by

$$L = \bar{\psi}_x G^{-1}_{xy} \psi_y = \bar{\psi}_x (i \not{\partial} - m)\psi_y \tag{2.4}$$

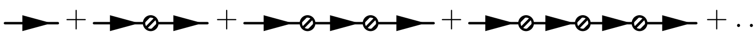
We get the normal free particle propagator by inverting this,

$$G = \frac{\not{p} + m}{p^2 - m^2} \tag{2.5}$$

Another way to understand this is to think of the mass term as an interaction term, describing the interaction

$$-m\bar{\psi}\psi =$$  . (2.6)

A particle propagating from a point to another can interact with itself any number of times on its way. If S is the free massles propagator $\not{p}/p^2$, the two-point propagator is an infinite sum

$G =$  (2.7)

$$\begin{aligned} &= S + SmS + SmSmS + \cdots = S(1 + mS + (mS)^2 + \dots) \\ &= \frac{S}{1 - mS} = \frac{\not{p} + m}{p^2 - m^2}. \end{aligned}$$

In this way mass can be understood as a self-energy, an energy coming from the particles interaction with itself. We will later see that a particle in hot plasma will have a self energy, due to its interaction with the plasma, and that this self energy can as well be understood as mass.

Normal fermion fields can be divided into two chiral states, the left-handed component $\psi_L = P_L\psi = \frac{1-\gamma_5}{2}\psi$ and the right-handed component $\psi_R = P_R\psi = \frac{1+\gamma_5}{2}\psi$. γ_5 is the chirality matrix $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. We can quite easily see that $\bar{\psi}_{L,R} = \bar{\psi}P_{R,L}$ and that $(\psi_{L,R})^c = P_{R,L}(\psi^c)$, where the ^{c} denotes charge conjugation. It is often said, though not exactly true, that charge conjugation changes the parity of the particle. The free particle lagrangian can be written using the chiral states as

$$\begin{aligned} L &= \bar{\psi}_L i \not{\partial}\psi_L + \bar{\psi}_R i \not{\partial}\psi_R - m\bar{\psi}_L\psi_R - m\bar{\psi}_R\psi_L \\ &= \bar{\psi}_L i \not{\partial}\psi_L - m\bar{\psi}_L\psi_R + h.c \end{aligned} \tag{2.8}$$

We see that for massive particles the chiral states are mixed as one can turn into the other. If understood as an interaction, this makes it possible for a particle to change its chirality. For a massless particle the chiral states do not get mixed and remain completely independent.

The Higgs model explains mass in as an interaction. The Higgs field is a scalar field that, because of the form of its potential, has a nonzero vacuum expectation value. Particles can couple to the field trough the lagrangian

$$L = \bar{\psi}i \not{\partial}\psi - \lambda\bar{\psi}_L\phi\psi_R + h.c \tag{2.9}$$

At small temperatures the Higgs field ϕ can be replaced by its expectation value, effectively giving the particle a mass term. The Higgs field also breaks the electro-weak symmetry giving W and Z bosons their mass and creating the electromagnetic and weak interactions. For a charged particle the mass term in 2.9 is the only possible mass term. This form of mass is called the Dirac mass. A particle with no charge, however, can have another kind of mass, the Majorana mass. It is given by the term $-M\bar{\psi}^c_R\psi_L - M\bar{\psi}^c_L\psi_R$ in the lagrangian. Here $\psi^c_{L,R} = (\psi^c)_{L,R}$ and we denote Majorana masses by M instead of m. The full free particle lagrangian of a chargeless particle with Majorana and Dirac masses can be written as

$$L_m = -\begin{pmatrix} \bar{\psi}^c_R & \bar{\psi}_R \end{pmatrix} \begin{pmatrix} M_1 & m \\ m & M_2 \end{pmatrix} \begin{pmatrix} \psi_L \\ \psi^c_L \end{pmatrix} \tag{2.10}$$

If the particle now had a charge, the Majorana term would permit an interaction that, turning a particle into its antiparticle, would violate charge conservation. This is why only a neutral particle, such as the neutrino, can have a Majorana mass.

2.3 The standard model and neutrino mass

The standard model can be divided phenomenologically into two parts. The strong sector describes the strong force and particles interacting trough it. As the strong force is, correctly named, much stronger than the other forces, it usually dominates over electroweak forces, which can then be neglected. The electro-weak sector, which we are now more interested in, describes electroweak interaction and Higgs interactions.

Table 2.1: Charges of the Electroweak sector particles.

	ν	l_L	N	l_R	Φ^+	Φ^0
T^3	$1/2$	$-1/2$	0	0	$1/2$	$-1/2$
Y	$-1/2$	$-1/2$	0	-1	$1/2$	$1/2$
$Q = Y + T^3$	0	-1	0	-1	1	0

When the electroweak symmetry is unbroken, we have two symmetries, a U(1)-symmetry with the associated weak hypercharge Y and a SU(2)-symmetry with the charge called the weak isospin T^3. The electric charge of a particle is given by $Q = T^3 + Y$ in the units of the electron charge. Left-handed particles form doublets that interact through the SU(2) interaction and have isospin $T^3 = \pm 1/2$. Right handed particles form singlets, don't interact trough SU(2) and have $T^3 = 0$. The charges of leptons are given in table 2.1. The lepton doublets are

$$l = \begin{pmatrix} \nu_e & \nu_\mu & \nu_\tau \\ e^-_L & \mu^-_L & \tau^-_L \end{pmatrix}$$

Here ν is always left handed, since there are no right-handed neutrinos in the standard model. The right-handed particles are singlets and don't interact through the SU(2) interaction. The lepton singlets are

$$r = \begin{pmatrix} e^-_R & \mu^-_R & \tau^-_R \end{pmatrix}$$

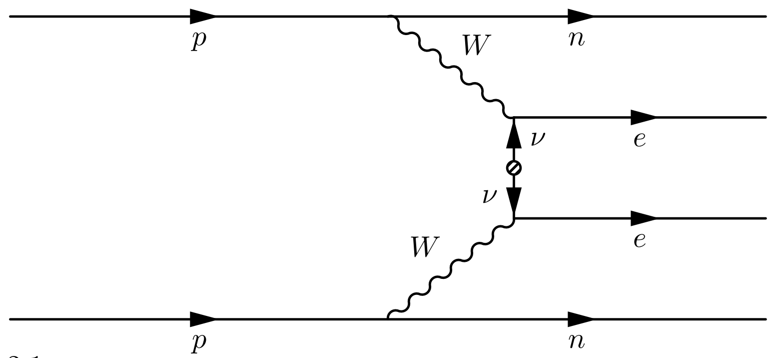


Figure 2.1: Neutrinoless double beta decay made possible by the Majorana mass term.

The Higgs field is a doublet

$$\Phi = \begin{pmatrix} \Phi^+ \\ \Phi^0 \end{pmatrix}$$

Using the Higgs doublets as in equation 2.9 the mass terms can be written as

$$L_m = -F_{ij}\sqrt{2}\bar{r}_i\Phi^\dagger l_j - F_{ij}\sqrt{2}\bar{l}_i\Phi r_j \tag{2.11}$$

F_{ij} are the Yukawa couplings and determine the mass relative to other particles. They are, in general, complex numbers, and it is their complex phases that cause CP violations within the standard model. The $\sqrt{2}$ is added because the potential of the Higgs doublet is usually taken to be

$$V(\psi) = m^2|\psi|^2 + \lambda|\psi|^4 \tag{2.12}$$

If m^2 is negative, this has a nonzero minimum. The expectation value can be taken as

$$\Phi = \begin{pmatrix} 0 \\ \sqrt{\frac{-m^2}{2\lambda}} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}.$$

This way, the mass of the lepton i is $F_{ii}v$, with $v \approx 246$GeV.

Now we know that neutrinos are not actually massless. The only way to have massive neutrinos without adding right-handed neutrinos would be to give them a Majorana mass $-M\bar{\nu}^c\nu$. This, however, would allow a process called the neutrinoless double beta decay, depicted in the figure 2.1. As this process hasn't been observed, the Majorana mass has to be very small, too small to explain the neutrino oscillations. We will here take it to be zero, as it is very small compared to the other mass terms. So to explain the oscillations, neutrinos have to have a Dirac mass, and right-handed neutrinos have to exist. The neutrino singlet can be denoted as

$$N = \begin{pmatrix} N_e & N_\mu & N_\tau \end{pmatrix}$$

So ν is a left-handed neutrino and N is a right-handed one. Note that in for example N^c we take the chirality operator to come first, so $N^c = (\nu_R)^c = (\nu^c)_L$. The antiparticle of N is actually left-handed. This notation is chosen because ν is traditionally always left-handed and ν^c right-handed. The neutrino singlets have all charges zero, they only interact through the mass term. This is why they are often called sterile neutrinos. The smallness of the neutrino masses can be explained by giving the sterile neutrinos a large Majorana mass. The mass term in the matrix form of eqn. 2.13 for a single flavor of neutrinos is

$$L_m = -\begin{pmatrix} \bar{\nu}_e & \bar{N}_e^c \end{pmatrix} \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} \nu_e \\ N_e^c \end{pmatrix} \tag{2.13}$$

To get the mass eigenstates, one has to diagonalise this matrix. This is straightforward and taking M to be much larger than m one gets the eigenstates

$$\begin{aligned} \nu_1 &\approx \nu_e + \frac{m}{M} N_e^c \\ N_1 &\approx \frac{m}{M}\nu_e^c + N_e \end{aligned} \tag{2.14}$$

and the eigenvalues

$$\begin{aligned} m_\nu &\approx -\frac{m^2}{M} \\ m_N &\approx M. \end{aligned} \tag{2.15}$$

The minus sign in the mass of ν_1 is taken as a complex phase in the Yukawa coupling, so that the actual mass is positive. There can always be complex phases in the mass matrix, they actually create the CP-violation, which is necessary for the creation of the lepton asymmetry. Now the ν_1 state is essentially the left-handed neutrino we see in experiments and N_1 the right-handed neutrino. With a large enough M the left-handed neutrino becomes light. This is called the see-saw mechanism. Both mass states include the other interaction state, but suppressed by the small factor m/M. The difference between the mass states and the interaction states of neutrinos can often be ignored.

The Dirac mass terms come from the Yukawa couplings of neutrinos to the Higgs boson. To do this, we must introduce a new form of the Higgs doublet,

$$\tilde{\Phi} = i\sigma_2\Phi = \begin{pmatrix} \Phi^{0\dagger} \\ -\Phi^- \end{pmatrix}$$

Now the mass term is $-\sqrt{2}H_{ij}\bar{N}_i\tilde{\Phi}^\dagger l_j - \sqrt{2}H_{ij}\bar{l}_i\tilde{\Phi}N_j$, where the H_{ij} are the Yukawa couplings. The Majorana mass can not be explained in the same way. Some ways of explaining it trough a similar interaction have been suggested. They involve introducing a new Higgs type particle or, in the case of νMSM, possibly coupling the right-handed neutrino with the inflaton, which is responsible for the early exponential expansion of the Universe. In any case, even at temperatures at which the electro-weak symmetry remains unbroken the right-handed neutrinos still have their Majorana mass. The whole free particle lagrangian of neutrinos is therefore

$$L = \bar{l}i \not{\partial} l + \bar{N}i \not{\partial} N - \sqrt{2} H_{ij} \bar{N}_i \tilde{\Phi}^\dagger l_j - \frac{1}{2} M_i \bar{N}_i^c N_i + h.c \tag{2.16}$$

Adding the Majorana mass to the right-handed neutrinos has some profound consequences for the theory. It breaks one of the most important symmetries in the standard model, the symmetry between particles and antiparticles. If a theory has a global symmetry, it always has a conserved current associated with it, and in this case the conserved current is the lepton number. As the symmetry is broken, lepton number is no longer conserved, meaning that interactions involving the right-handed neutrinos can violate it. The double beta decay in figure 2.1 is one example and it is actually made possible by the right-handed Majorana term. The interaction in the shaded blob is $\nu \to N \to N^c \to \nu^c$ The rate is suppressed by a coefficient $(m/M)^2$, but might still be observable. In double beta decay, the lepton number changes by two.

2.4 Parameters of the νMSM

We have basically introduced all the particles of the νMSM. All it adds to the standard model are the right-handed neutrinos with a Majorana mass. It also includes some limitations to the parameters arising from the requirements that the lightest of the sterile neutrinos works as dark matter and that the observed baryon asymmetry is explained.

We may now write the complete Lagrangian of the νMSM, using L_{SM} for the standard model Lagrangian and labeling the three generations with i and j

$$L = L_{\mathrm{SM}} + \bar{N}_i i \not{\partial} N_i - \sqrt{2} G_{ij} \bar{N}_i \tilde{\Phi}^\dagger l_j - \frac{1}{2} M_i \bar{N}_i^c N_i + h.c \tag{2.17}$$

There are altogether 18 new parameters in this lagrangian. There are three Majorana masses for the three right-handed neutrinos and 15 degrees of freedom in the Yukawa matrix. The latter ones can be taken to be 3 diagonal couplings, 6 mixing angles 6 complex phases. The whole matrix can be written as

$$G = K_L P_\alpha h_d K_r^\dagger P_\beta, \tag{2.18}$$

where $h_d = \text{diag}(h_1, h_2, h_3)$ includes the diagonal couplings, P matrices are diagonal matrices of Majorana phases

$$P_\alpha = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, 1), \quad P_\beta = \text{diag}(e^{i\beta_1}, e^{i\beta_2}, 1), \tag{2.19}$$

and the Kobayashi-Maskawa -type mixing matrix K_L is

$$K_L = \tag{2.20}$$
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{L23} & s_{L23} \\ 0 & -s_{L23} & c_{L23} \end{pmatrix} \begin{pmatrix} c_{L13} & 0 & s_{L13}e^{-i\delta_L} \\ 0 & 1 & 0 \\ -s_{L13}e^{i\delta_L} & 0 & c_{L13} \end{pmatrix} \begin{pmatrix} c_{L12} & s_{L12} & 0 \\ -s_{L13} & c_{L13} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Here we have used a short hand notation $c_{Lij} = \cos(\theta_{Lij})$ and $s_{Lij} = \sin(\theta_{Lij})$. K_R is exactly analogous, just replacing L with R. Thus all the 15 degrees of freedom are included in the couplings h or the angles α, β, θ or δ. One could get three extra terms as the Majorana masses for the left-handed neutrinos, but they would be very small and wouldn't affect the phenomenology.

The masses of the left-handed neutrinos limit these parameters slightly, but leave a lot of freedom. Because of the assumption that the lightest right-handed neutrino is the dark matter, its mass is strongly restricted [6]. Its Yukawa coupling has to be rather small to explain its long lifetime and the mass of the associated left-handed neutrino. For the Majorana mass cosmological measurements give

$$2keV \lesssim M_1 \lesssim 5keV \tag{2.21}$$

The lower bound is due to observations of the cosmic microwave background and spectral measurements [14]. The upper bound comes from the limit of X-ray observations to the decays of the dark matter particles [15]. In [6], the article introducing the theory, it was shown that the measurement constraints of the dark matter can only be satisfied if there are three or more right-handed neutrinos and the lightest of them is the dark matter. Given it is light enough, it will have an expected lifetime that exceeds the age of the Universe greatly. This way it can have a nonzero density even though it is decaying all the time.

Chapter 3

The baryon asymmetry

The standard model is highly symmetric with respect to changes between matter and antimatter. They have the same mass, same decay widths and exactly opposite charges. Within the standard model, it is therefore impossible to explain the fact that most of our Universe is consists of matter, and there is actually very little antimatter around. At least this is expected to be true, we are yet to encounter any large amounts of antimatter within the explored Universe, and cosmic rays from our own galaxy would suggest that the whole galaxy is made of matter. Some antimatter is produced as the particles scatter in the atmosphere and this is observed. The most compelling argument for baryon asymmetry is that it seems impossible to come up with a mechanism which would separate matter and antimatter in different regions so that they wouldn't be able to annihilate. These regions should be completely separated, otherwise we should observe radiation from the annihilations at the boundary.

One might argue that the Universe was born with more matter than antimatter. It is, however, important to look for other solution, as this might effect many other prediction about the early Universe. The inflation model, a widely accepted theory explaining many problems such as the flatness of the topology of our Universe, predicts that after the era of inflation the Universe was in a state completely dominated by radiation. There was no matter, and therefore no baryon asymmetry. There are several theories explaining the creation of the asymmetry from such a condition. For a more complete description of the problem and possible solutions see for example [13]. A good description of leptogenesis, the baryogenesis theory we are most interested in, can be found in [16].

The baryon asymmetry is actually rather small, on a cosmic scale, since most of the Universe is actually almost empty. Using the observed abundances of light elements and current theories of how they were formed, one ends up with the difference of baryon and antibaryon numbers of about 10^{-10}, normalized to the density of entropy.

To be able to do any quantitative analysis of the baryon asymmetry, we need to define proper variables describing it. Baryons are hadrons, composite particles of quarks that are held together by the strong force, that have three quarks of different colors in them. Though there are plenty of different kinds of baryons, only protons and neutrons are stable enough to be relevant to cosmology. All directly observable matter consist of protons, neutrons and, of course, electrons. Since no free quarks can be observed in temperatures relevant to our everyday lives, and baryons are so important, we count baryons and not quarks. The baryon number B is the quantum number assigned to all the quarks. Normal quarks have value $\frac{1}{3}$ and antiquarks $-\frac{1}{3}$, so that hadrons have baryon number 1 and antihadrons -1.

The amount of baryon number in a unit volume N_B does not remain constant as the Universe expands. It is convenient to use a variable that does and one possible choice is the density of baryons with respect to the radiation density of the Universe N_γ. We define

$$\eta = \frac{N_B}{N_\gamma}. \tag{3.1}$$

This parameter is important in determining the amounts of light baryons created during the nucleosynthesis and should remain constant at current temperatures. The photon number density N_γ at temperature T is

$$N_\gamma = 2\frac{\zeta(3)}{\pi^2}T^3, \tag{3.2}$$

where $\zeta(x)$ is the Riemann zeta function. We may estimate η as [17]

$$4 * 10^{-10} \geq \eta \geq 7 * 10^{-10} \tag{3.3}$$

Though this asymmetry is small, it is far too much to be explained by expected statistical fluctuations.

3.1 Sakharov's conditions

In 1967 Sakharov proposed a possible way of generating tiny amounts of baryon asymmetry within the standard model [18]. There are three conditions for this, and though the asymmetry produced is far too small, these conditions must hold for any theory describing baryogenesis, even outside the standard model.

The first condition is rather obvious. There must be baryon number violation. Initially the Universe is baryon symmetric, $B = 0$, and we want to end up with a baryon asymmetric Universe, $B \neq 0$. The baryon number must change somewhere on the way. Processes violating baryon number might also mediate proton decay and are constrained by the proton lifetime, which is longer than 5×10^{32} years. There are processes called sphalerons in the standard model that violate the baryon number but keep $B - L$, baryon number minus lepton number, constant. These are described in section 3.2.

The second condition is that there must be violation in symmetries in C, charge conjugation, and CP, product of charge and parity conjugation. In a system conserving C the processes $i \to f$ and $\bar{i} \to \bar{f}$ would be as probable. If the first process were to increase baryon number by any amount, the latter would decrease it by the same amount, and no net change would accumulate. Because CPT, where T stands for time reversal, should always be conserved in a field theory, CP conservation implies T conservation. Therefore, without CP-violation, the process

$$i(\mathbf{r}_i, \mathbf{p}_i, \mathbf{s}_i) \to f(\mathbf{r}_f, \mathbf{p}_f, \mathbf{s}_f) \tag{3.4}$$

would be as likely as the time reversed one

$$f(\mathbf{r}_f, -\mathbf{p}_f, -\mathbf{s}_f) \to i(\mathbf{r}_i, -\mathbf{p}_i, -\mathbf{s}_i). \tag{3.5}$$

If now the first process would create baryons of at some point in the phase space, the second would destroy the same amount at another point, and there would be no net change. The C symmetry is violated by the weak interaction and the CP symmetry can be violated trough complex angles in the in the mass matrix.

The third condition is departure from thermal equilibrium. At equilibrium in a CPT-conserving theory the expectation value for the baryon number is $\langle B \rangle = 0$. We can easily show this using the facts that the hamiltonian is invariant and the baryon number operator is odd in CPT-transformation.

$$\begin{aligned} \langle B \rangle &= Tr[e^{-\beta \hat{H}} \hat{B}] \qquad (3.6) \\ &= Tr[(CPT)^{-1}(CPT) e^{-\beta \hat{H}} \hat{B}] \\ &= Tr[(CPT) e^{-\beta \hat{H}} \hat{B} (CPT)^{-1}] \\ &= Tr[e^{-\beta \hat{H}} (CPT) \hat{B} (CPT)^{-1}] \\ &= -Tr[e^{-\beta \hat{H}} \hat{B}] = -\langle B \rangle \end{aligned}$$

As CPT invariance seems to be necessary for field theories, processes producing the excess baryon number cannot take place in thermal equilibrium. This condition is achieved in the νMSM by the decay of the massive right-handed neutrinos. The Universe expands too rapidly for the resulting particles to equilibrate. As there should be enough of the lightest right-handed neutrinos to account for dark matter, it is mainly the decays of the two other generations of them that generate this effect.

3.2 The Sphaleron

The most general Lagrangian that is invariant under the standard model gauge group SU(3)$_C\times$SU(2)$_L\times$U(1)$_Y$, assuming that the Higgs particle does not carry color charge, is invariant under the leptonic and baryonic transformations. What this means is that both lepton and baryon number are conserved at any level of the perturbation theory. These are accidental symmetries that are not required by the theory. However, the perturbative expansion can fail to describe all aspects of the theory. In this case, in 1976, 't Hooft [24] showed that there are nonperturbative processes violating $B+L$, while keeping $B-L$ conserved. This essentially turns excess lepton number to baryon number. The probability of these processes are small in normal temperatures, but was large enough in the high temperatures of the early Universe [25]. A good description of this effect, including some calculations, is given in [13].

As always in field theories, for all symmetries there is a conserved current. The current associated to the baryonic and leptonic symmetries are baryon and lepton number. Because of effects not present in perturbation theory, the currents are actually not conserved. With N_G as the number of generations of fermions, F and f the gauge fields of SU(2)$_L$ and U(1)$_Y$ respectively, with g and g' coupling constants, the divergences of the baryonic and leptonic currents are

$$\partial_\mu J^\mu_B = \partial_\mu J^\mu_L = i\frac{N_G}{32\pi^2}\left(-g^2 F^{a\mu\nu}\tilde{F}^a_{\mu\nu} + g'^2 f^{\mu\nu}\tilde{f}_{\mu\nu}\right) \tag{3.7}$$

Here we have a dual field $\tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\sigma\rho}F^{\sigma\rho}$.

As the divergences are the same, $B-L$ will still be conserved. This is actually true for al leptonic generations separately, $\frac{1}{3}B - L_i$, where i counts the generations, is conserved. Nevertheless, all the generations of particles have to be present in the reaction and both baryon and lepton number change in multiples of three. This gives rise to an operator of the form

$$\prod_{i=1}^{3}(q_i q_i q_i l_i), \tag{3.8}$$

where i runs over generations. The operator involves 12 fermions. This is the reason the process is highly unlikely in normal temperatures. In higher temperatures, however, in the particle plasma such collisions are possible. When the rate of the processes is greater than the expansion rate of the Universe, the sphaleron processes are at thermal equilibrium. Using this fact we can calculate how much of the lepton number is translated to baryon number [12]. We are only interested in temperatures between the sphaleron freeze out temperature $T_{\rm EW} \approx 100$GeV and 10^{13}GeV, above which the sphalerons are too slow to stay in equilibrium.

Above $T_{\rm EW}$ all particles except the Higgs boson and the right-handed, or sterile, neutrinos are massless. The rates of the Yukawa interactions not involving neutrinos are suppressed only by the Higgs mass and are fast enough to stay at euilibrium. These processes are

$$\begin{aligned} q_i &\leftrightarrow \tilde{\Psi} u_j \\ q_i &\leftrightarrow \Psi d_j \\ l_i &\leftrightarrow \Psi e_j \end{aligned} \tag{3.9}$$

The lepton doublets and singlets were defined in 2.3 and the quark doublets and singlets are exactly analogous, with the left-handed doublet q_i and the right-handed singlets u_i and d_i. i labels the generation. The quark doublet is connected to the up-singlet u_j by $\tilde{\Psi} = i\sigma_2\Psi$, the same doublet we used to connect the lepton doublet with right-handed neutrinos. It has the chemical potential $\mu_{\tilde{\Psi}} = -\mu_\Psi$. As the processes are in equilibrium, the chemical potentials of the particles on the left-hand sides are equal to the ones of those on the right-hand sides,

$$\begin{aligned} \mu_{q_i} + \mu_\Psi - \mu_{u_j} &= 0 \\ \mu_{q_i} - \mu_\Psi - \mu_{d_j} &= 0 \\ \mu_{l_i} - \mu_\Psi - \mu_{e_j} &= 0. \end{aligned} \tag{3.10}$$

Since the particles can change generations by continuing these processes, for example $q_i \to \tilde{\psi} u_j \to q_k$, the chemical potentials for different generations are the same, for all i, j

$$\begin{aligned} \mu_{q_i} = \mu_{q_j} &= \mu_q \\ \mu_{l_i} = \mu_{l_j} &= \mu_l \\ \mu_{u_i} = \mu_{u_j} &= \mu_u \\ \mu_{d_i} = \mu_{d_j} &= \mu_d \\ \mu_{e_i} = \mu_{e_j} &= \mu_e. \end{aligned} \tag{3.11}$$

We can also assume that the sphalerons are at equilibrium. The process in equation 3.8 gives

$$\sum_i (3\mu_{q_i} + \mu_{l_i}) = 0 \Rightarrow 3\mu_q + \mu_l = 0 \tag{3.12}$$

We still need one equation. Since the electric charge is conserved, and the Universe seems to be electrically neutral today, we assume it was electrically neutral also at the time the baryon asymmetry was generated. Assuming that the gauge bosons contribute no net charge, we have for the charge density

$$\rho_Q = \sum q(n-\bar{n}) \tag{3.13}$$
$$= \sum_i \left(\frac{1}{3}(n_{q_i} - \bar{n}_{q_i}) + \frac{2}{3}(n_{u_i} - \bar{n}_{u_i}) - \frac{1}{3}(n_{d_i} - \bar{n}_{d_i}) - (n_{l_i} - \bar{n}_{l_i}) - (n_{e_i} - \bar{n}_{e_i}) \right)$$
$$+ (n_\Psi - \bar{n}_\Psi) = 0$$

The number densities n are related to the chemical potentials trough

$$n_f - \bar{n}_f = \int_0^\infty \frac{d\mathbf{p}^3}{(2\pi)^3} \frac{g(p)}{e^{(p-\mu)/T}+1} - \frac{g(p)}{e^{(p+\mu)/T}+1} \tag{3.14}$$

for fermions. As the temperature is high, μ/T is small, and we can expand around $\mu/T = 0$. The zeroth order term naturally disappears and we have in the first order

$$n_f - \bar{n}_f = \frac{2g\mu}{T} \int_0^\infty \frac{dp}{(2\pi)^2} \frac{p^2 e^{p/T}}{(e^{p/T}+1)^2} = \frac{gT^2\mu}{12}, \tag{3.15}$$

where g is the number of degrees of freedom. For bosons we get similarly

$$n_b - \bar{n}_b = \frac{2gT^2\mu}{12} \tag{3.16}$$

For quarks $g = 6$ and other fermions and bosons $g = 2$. Using these and 3.13 we have

$$\mu_q + 2\mu_u - \mu_d - \mu_l - \mu_e + \frac{2}{N_g}\mu_\Psi = 0, \tag{3.17}$$

where N_g is the number of generations. We now have a group of five equations

$$\begin{aligned} \mu_q + \mu_\Psi - \mu_u &= 0 \\ \mu_q - \mu_\Psi - \mu_d &= 0 \\ \mu_l - \mu_\Psi - \mu_e &= 0 \\ 3\mu_q + \mu_l &= 0 \\ \mu_q + 2\mu_u - \mu_d - \mu_l - \mu_e + \frac{2}{N_g}\mu_\Psi &= 0. \end{aligned} \tag{3.18}$$

From these we can solve for all the other chemical potentials in terms of, for example, μ_q

$$\begin{aligned}
\mu_l &= -3\mu_q \\
\mu_\Psi &= -\frac{4N_g}{1+2N_g}\mu_q \\
\mu_e &= -\frac{3+2N_g}{1+2N_g}\mu_q \\
\mu_u &= \frac{1-2N_g}{1+2N_g}\mu_q \\
\mu_d &= \frac{1+6N_g}{1+2N_g}\mu_q
\end{aligned} \tag{3.19}$$

We can then write chemical potentials for lepton and baryon number

$$\begin{aligned}
\mu_B &= N_g\frac{2\mu_q+\mu_u+\mu_l}{2} = 2N_g\mu_q \\
\mu_L &= N_g\frac{2\mu_l+\mu_e}{2} = -\frac{9+14N_g}{2+4N_g}N_g\mu_q
\end{aligned} \tag{3.20}$$

From these we have, taking $N_g = 3$

$$B = \frac{4+8N_g}{13+22N_g}(B-L) = \frac{28}{79}(B-L) \tag{3.21}$$

If the phalerons, and all the aforementioned processes, are in equilibrium, this is the amount of the total asymmetry that is transferred to the baryon sector.

3.3 Theories of Baryogenesis

Many models have been proposed to explain baryogenesis. Most of these fall into one of three categories, electroweak baryogenesis, GUT baryogenesis and leptogenesis. Electroweak baryogenesis is the process proposed by Sakharov and doesn't require any extension to the standard model. All the other possibilities require at least some changes to it. Electroweak- and GUT baryogenesis are described in detail in [13].

Even the standard model alone can fulfill Sakharov's conditions. The electroweak interaction violates the C-symmetry and the the complex angles in the mass mixing matrix violate the CP-symmetry. Baryon number is violated in non-perturbative standard model in sphalerons. Departure from thermal equilibrium can be achieved through phase transitions, such as the electroweak transition where the Higgs particle gets its vacuum expectation value, or the decay of very massive weakly interacting particles.

The CP-violation in the standard model must however involve all three generations of leptons, which makes it rather small at temperatures below the electroweak transition. The baryon asymmetry generated within the standard model remains therefore far too small to account for the observed abundance of matter in the Universe. This is considered as evidence of physics beyond the standard model.

The standard model consists of two separate symmetry groups, SU(3) for QCD and SU(2)×U(1) for electroweak interactions. GUTs (Grand Unified Theories) are theories that try to explain all interactions using just one larger symmetry group, including these as its subgroups. The normal standard model interactions appear as this symmetry is spontaneously broken. Many such theories have been formed, though no single theory has been completely accepted.

It is easy to fulfill the Sakharov's conditions within these models. They involve baryon and lepton number violations and massive particles quite naturally. CP-violation is produced through complex couplings in their decays. Baryogenesis can therefore be explained quite easily within a GUT. The problem is formulating a theory which is consistent with the standard model and the experimental upper limit of the proton lifetime. The GUT baryogenesis should take place during or after the reheating and before the GUT symmetry breaking. This is a strict limit to reheating temperatures, as the GUT scale is large, often $\sim 10^{15}$GeV

The baryogenesis of the νMSM falls into the category of leptogenesis. In leptogenesis, as in EW baryogenesis, baryon number is violated through sphaleron processes, turning excess lepton number to baryon number. As the sphalerons are in equilibrium at temperatures 100GeV$< T < 10^{13}$GeV, the excess lepton number needs to be generated at rather high temperatures. Usually the lepton number is generated through the decays of massive particles, in our case the right-handed neutrinos.

The electroweak sector violates C maximally. In leptogenesis, the CP-violation is in the complex Yukawa couplings of the massive particles. The Yukawa couplings are taken to be complex in such a way, that all the complex angles cannot be absorbed in the definitions of the particle fields. This way, CP is violated by the decays of the particles. Thermal equilibrium is again broken by the decays of weakly interacting but massive particles. At some high temperature they are at equilibrium, but when the temperature reaches their mass, their equilibrium density drops suddenly, and they start to decay out of equilibrium. This works as the source of lepton number, which is then transformed to baryon number.

In νMSM leptogenesis this happens slightly differently. The heavy particles, right-handed neutrinos, interact too weakly to reach equilibrium. Their abundance is close to zero in the beginning and starts to approach equilibrium value. Due to this departure from equilibrium of the right-handed neutrinos, an excess left-handed ones is generated.

So after inflation and reheating the right-handed neutrino density is below equilibrium. The inflaton decays into other particles taking their density at least close to equilibrium value. Lepton number violating scatterings wash out any excess of leptons or baryons generated at this point. A critical temperature for νMSM is $T \sim (\Delta M_{32}^2 M^0)^{1/3}$, ΔM_{32}^2 being square difference of the Majorana masses of the two heavier neutrinos and $M^0 = 7 \times 10^{-17}$GeV appears in the time temperature relation

$$t = \frac{M^0}{2T^2} \tag{3.22}$$

At this temperature the standard model neutrinos decay into right-handed neutrinos. They then start to decay back to standard model neutrinos. These decays violate CP and produce more leptons than antileptons. Below this temperature the produced asymmetry works as the source of a slowly increasing asymmetry of the right-handed neutrinos and this asymmetry gets translated back to standard model neutrinos. When $T < 10^{13}$GeV sphaleron processes convert lepton number to baryon number. At smaller temperatures the sphalerons are fast compared to the lepton number violating decays and scatterings and are able to keep the baryon and lepton numbers in equilibrium. Around the electroweak scale $T \sim T_{\rm EW}$, fermions acquire mass as the electroweak symmetry breaks and the Higgs field acquires a vacuum expectation value. The sphaleron processes also freeze out at the same scale, leaving baryon number to its contemporary value. Sphaleron processes become greatly suppressed and baryon number is virtually conserved.

Chapter 4

Calculation in finite temperature

The baryon asymmetry developed in the early history of the Universe, when it was still very hot and dense, filled only with particle plasma. All the processes took place in this plasma. This, of course, had a great effect to the process. Particles interacted with those of the plasma, slowing them down and effectively making them more massive. They also decayed by scattering with other particles of the plasma.

In principle, plasma is made of particles and can be described by quantum field theory, but it would of course be impossible in practice to take all the particles into account. Instead, a set of Feynman rules can be derived that incorporates the average effects of the plasma, using both field theory and thermodynamics. We will not derive these rules, just introduce them briefly. There is a lot of literature on the subject, for example references [19] or [21].

In classical thermodynamics, the probabilities of all the possible states with energy E_n at inverse temperature $\beta = 1/T$ are

$$P(E_n) = \frac{e^{-\beta E_n}}{Z(\beta)}, \tag{4.1}$$

where the normalization factor $Z(\beta)$ is the partition function

$$Z(\beta) = Tr\left(e^{-\beta \hat{H}}\right) = \sum_n g_n e^{-\beta E_n}, \tag{4.2}$$

where g_n is the degeneracy of state n. The energies E_n are of course eigenvalues of hamiltonian operator $\hat{H}$ and the sum goes over all possible states. We can rewrite the partition function using a complete set of eigenvectors $|q\rangle$ of, say, position.

$$Z(\beta) = \int dq \langle q|\, e^{-\beta \hat{H}}\, |q\rangle. \tag{4.3}$$

In quantum mechanics, the probability of a state q at time t evolving to the state q' at time t' in the operator formalism this is

$$F(q', t'; q, t) = \langle q(t')|\, e^{-i\hat{H}(t'-t)}\, |q(t)\rangle \tag{4.4}$$

This can also be written in the path integral formalism

$$F(q', t'; q, t) = \int Dq(t")e^{i\int_t^{t'} dt" H(q(t"))} \tag{4.5}$$

The integral is taken over all paths with $q(t') = q'$ and $q(t) = q$. These formalisms can be shown to be completely equivalent. In the operator formalism the time evolution and the partition function 4.3 are quite similar,

$$Z(\beta) = \int dqF(q, -i\beta; q, 0) \tag{4.6}$$

We can formally define a path integral representation for the partition function

$$Z(\beta) = \int Dq(t)e^{i\int_0^{-i\beta} dtH(q(t))} = \int Dq(t)e^{iS(-i\beta)} \tag{4.7}$$

Here the integral goes over all paths with boundary values $q(\beta) = q(0)$, meaning that the paths have period β in imaginary time. We can now define a generating functional just as in quantum field theory,

$$Z(-i\beta; j) = \int Dq(t)e^{iS(-i\beta)}\hat{T}\left[e^{\int_0^{-i\beta} j(t)q(t)dt}\right] \tag{4.8}$$

$\hat{T}$ is here the time ordering operator, defined in imaginary time. In normal field theory differentiating the generating functional twice and taking $j = 0$ would give us the two-point propagator. Doing this we have

$$\begin{aligned} &\frac{1}{Z(-i\beta; j)} \frac{d^2 Z(-i\beta; j)}{dj(-i\beta_1)\, dj(-i\beta_2)}\bigg|_{j=0} \\ &= \frac{1}{Z(\beta)} \int Dq(t)\hat{T}\left[q(-i\beta_1)q(-i\beta_2)\right] e^{iS(-i\beta)} \end{aligned} \tag{4.9}$$

The right-hand side now is thermal average of a time ordered product of the two position operators $\hat{q}(i\beta_1$ and $\hat{q}(i\beta_2)$. We may now define this to be the propagator in imaginary time

$$\Delta(i\beta) = \frac{1}{Z(i\beta; j)} \frac{d^2 Z(i\beta; j)}{dj(i\beta_1)\, dj(i\beta_2)}\bigg|_{j=0} = \langle \hat{T}(\hat{q}(i\beta)\hat{q}(0))\rangle \tag{4.10}$$

This way we can calculate thermal averages using the formalism of field theories. We are of course actually interested in the propagators in real time and giving time an imaginary component, $t = t_{real} + i\beta$, we have their thermal averages in temperature $T = 1/\beta$. There are now two equivalent approaches to defining the finite temperature propagators. The mathematically simpler one is to use only imaginary time and simply take the Fourier transform of 4.10. Analytically continuing to real time leads to the Matsubara, or imaginary-time, propagator. Choosing the proper continuation this gives the usual zero temperature propagators with $\beta \to \infty$.

A possibly more intuitive, though mathematically more complicated, approach is to define the time integration along a new path on the complex plane that contains the real axis. This way the propagators have real time arguments. If the path begins at initial time t_i, it has to end at time $t_i - i\beta$. The time ordering operator then has to be defined along the chosen path. A common choice is to take $t_i < 0$ and integrate along the path $t_i \to -t_i \to -t_i - i\sigma \to t_i - i\sigma \to t_i - i\beta$, shown in figure 4.1. t_i is taken to be very small, approaching negative infinity. The generating functional can now be defined:

$$Z(-i\beta; j) = \int Dq(t) e^{iS(-i\beta)} T_C \left[e^{\int_C j(t) q(t) dt} \right] \qquad (4.11)$$

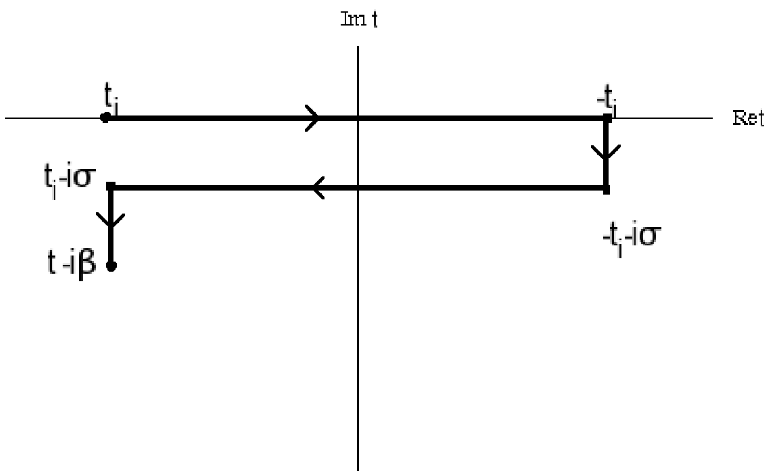


Figure 4.1: The time integration contour in real time formalism

Though the starting and ending points of both paths are the same, the time-ordering is different and gives a different form to the propagators. Both propagators are completely valid and give the same physical results. We will use the propagators derived using the latter, real-time, formalism. For our simple diagrams it is the easier and more descriptive one. For any more complicated calculations the imaginary-time formalism would be more effective.

The main advantage of the real-time formalism is that we can describe normal fields with real time values, but there is also an extra complication:

in addition to the normal fields there are also 'ghost' fields with time values $t - i\sigma$ and reversed time ordering. These correspond to excitations in the plasma and are truly physical phenomena, they have to be taken into account in order to get correct results. This means that there are two kinds of vertices, normal, or type 1, vertices and ghost, or type 2, vertices. These are connected with four different kinds of propagators. Two type 1 vertices are connected with S_{11}, two type 2 vertices with S_{22} and type 1 and type 2 vertices with S_{12} or S_{21}. To get the amplitude of a Feynman diagram on has calculate the amplitudes for all possible configurations of type 1 and 2 vertices and add them up. All external legs of course correspond to physical particles and are therefore attached to type 1 vertices. This is the reason calculations in real-time formalism get complicated with large diagrams.

The propagators for fermions in a contour in which we choose $\sigma = 0$ are now [19]

$$\begin{aligned} S_{11}(p) &= (\not{p} + m)\left[\frac{1}{p^2 - m^2 + i\epsilon} + i2\pi\delta(p^2 - m^2)n_f(\omega_p)\right] \\ S_{22}(p) &= (\not{p} + m)\left[\frac{1}{p^2 - m^2 + i\epsilon} - i2\pi\delta(p^2 - m^2)n_f(\omega_p)\right] \\ S_{12}(p) &= -2\pi(\not{p} + m)\left[\theta(p^0) - \theta(-p^0)\right] e^{\beta\omega_p/2} n_f(\omega_p)\delta(p^2 - m^2) \\ S_{21}(p) &= S_{12}(p) \end{aligned} \tag{4.12}$$

For gauge bosons we have the propagators

$$\begin{aligned} D^{11}_{\mu\nu}(p) &= -g_{\mu\nu}\left[\frac{1}{p^2 - m^2 + i\epsilon} - i2\pi\delta(p^2 - m^2)n_b(\omega_p)\right] \\ D^{22}_{\mu\nu}(p) &= -g_{\mu\nu}\left[\frac{1}{p^2 - m^2 + i\epsilon} + i2\pi\delta(p^2 - m^2)n_b(\omega_p)\right] \\ D^{12}_{\mu\nu}(p) &= -g_{\mu\nu} 2\pi e^{\beta\omega_p/2} n_b(\omega_p)\delta(p^2 - m^2) \\ D^{21}_{\mu\nu}(p) &= D^{12}_{\mu\nu}(p) \end{aligned} \tag{4.13}$$

Here $n_f(p) = \dfrac{1}{e^{\beta\omega} + 1}$ and $n_b = \dfrac{1}{e^{\beta\omega} - 1}$

For scalar bosons, such as the Higgs particle, the propagator is almost the same as for gauge bosons, except that we don't get the $-g_{\mu\nu}$. For example

$$D^{11}(p) = \frac{1}{p^2 - m^2 + i\epsilon} - i2\pi\delta(p^2 - m^2)n_b(\omega_p). \tag{4.14}$$

Both type 1 and type 2 vertices have the same values as in the normal zero temperature theory, only the propagators change. For convenience we write them down here. The terms in the lagrangian describing weak hypercharge is $\bar{\phi} i g' \gamma^\mu A \phi$, where ϕ is a fermion with weak hypercharge g' and A is the corresponding gauge boson. The lagrangian for the weak isospin is

$\bar{\phi}ig\gamma^\mu W\phi$, where g is the charge and W the gauge boson. In the Feynman diagrams the values of the vertices are the coefficients in the interaction term. These are

$$v_{g'} = ig'\gamma^\mu \quad \text{and} \quad v_g = ig\gamma^\mu \tag{4.15}$$

In addition to these there are the Yukawa couplings. These couple all the generations of particles together and are written as matrices. The matrix coupling the lepton and Higgs doublets to the right-handed neutrinos is G_{ij} and the one coupling them to the singlet leptons is F_{ij}. As the interactions are exactly similar for all generation, we will use f instead of the terms of these matrices in calculations. This makes them as general as possible. Later, when the particle types and generations are identified, f is replaced by the correct term in the matrices. The lagrangian terms are then $\sqrt{2}f\bar{\phi}\Psi\phi$, where again we are considering the interaction of a fermion ϕ. The vertices now have the value

$$v_f = \sqrt{2}f \tag{4.16}$$

In our calculations all the in section 4.2 all vertices are connected to external legs and have to be of type one. This simplifies the calculation greatly.

4.1 Some free energy diagrams and thermal mass

At high temperature, particles can acquire an effective mass. Interactions in which the outgoing particle is of the same kind as the ingoing resemble the interaction made possible by the mass term. We will now calculate the effective mass of an active neutrino in high temperature plasma. The calculation follows that in the article by H.A. Weldon [22].

We will calculate the selfenergy Σ depicted in the diagram 4.2 a), representing the process $\nu \to \phi l \to \nu$. This interaction is possible at high temperatures since the Higgs does not have its expectation value, but behaves as a normal SU(2) doublet. The integral itself is Lorentz independent and we work in the rest frame of the plasma, so the four-velocity of the plasma is $u = (1, 0, 0, 0)$. We will use K for the particles four-momentum and P for the loop four-momentum. Three-momenta will be marked with $\mathbf{k}$ and $\mathbf{p}$, respectively. The lengths of the momenta will be $k = |\mathbf{k}|$ and $p = |\mathbf{p}|$. We will take the Higgs- and sterile neutrino masses to be zero.

For the vertices we have $\sqrt{2}f$ and $\sqrt{2}f^*$. The selfenergy is thus

$$\Sigma(K) = i2|f|^2 \int \frac{d^4p}{(2\pi)^4} D(p)S(p+K) \tag{4.17}$$

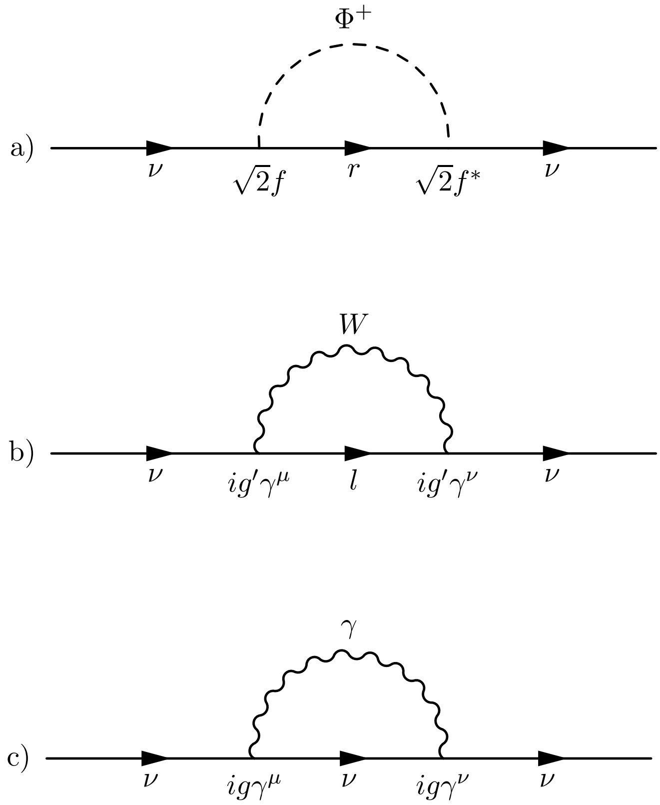


Figure 4.2: Self energy Feynmann diagrams of a neutrino in a high temperature plasma. In b), l is the corresponding left-handed lepton, for example for ν_e, $l = e_L$ and W the gauge boson of the SU(2) interaction. In c), γ is the U(1) gauge boson.

Using the real time propagators 4.12 and 4.14 we can write 4.17 as

$$\begin{aligned}\Sigma(K) &= \Sigma_{T=0}(K) + 2|f|^2 \int \frac{d^4p}{(2\pi)^4} \\ &\times \left(2\pi\delta(P^2) n_b(P_0) \frac{\not{P} + \not{K}}{(P+K)^2} - 2\pi\delta((P+K)^2) n_f(P_0+K_0) \frac{\not{P}+\not{K}}{P^2} \right) \\ &+ i2|f|^2 \int \frac{d^4p}{(2\pi)^4} (2\pi)^2 \delta(P^2)\delta((P+K)^2) n_b(P_0) n_f(P_0+K_0)(\not{P}+\not{K})\end{aligned}$$

We are now interested in the high temperature approximation of the self energy. The first term is just the temperature independent part of it. As we will later see, the leading order in temperature is T^2. We will therefore ignore the first term, and any arising term that is of lower order than T^2. Using a translation $P \to -P-K$ we can write the second part of the second term as

$$+2\pi\delta(P^2)n_f(P_0)\frac{\not{P}}{(P+K)^2}$$

Now we can use

$$\delta(P^2) = \delta(P_0^2 - p^2) = \frac{(\delta(P_0 - p) + \delta(P_0 + p))}{2p} \tag{4.18}$$

Writing $P^+ = (p, \mathbf{p})$ and $P^- = (-p, \mathbf{p})$ we get

$$\begin{aligned}
\Sigma(K) &= 2|f|^2 \int \frac{d^3p}{(2\pi)^3}\frac{1}{2p}\left(n_b(p)\frac{\not{P}^+ + \not{K}}{(P^+ + K)^2} + n_b(p)\frac{\not{P}^- + \not{K}}{(P^- + K)^2}\right) \qquad (4.19)\\
&+ 2|f|^2 \int \frac{d^3p}{(2\pi)^3}\frac{1}{2p}\left(n_f(p)\frac{\not{P}^+}{(P^+ + K)^2} + n_f(p)\frac{\not{P}^-}{(P^- + K)^2}\right)\\
&+ i2|f|^2 \int \frac{d^3p}{(2\pi)^2}\frac{1}{2p}[\delta((P_+ + K)^2)(\not{P}_+ + \not{K})n_b(p)n_f(p)\\
&\quad + \delta((P_- + K)^2)(\not{P}_- + \not{K})n_b(p)n_f(p)]
\end{aligned}$$

We expect the self-energy to depend only on the two natural Lorentz frames, the four-velocity of the plasma u and the four momentum of the particle K. Let us first calculate the real part and write $Re(\Sigma(K)) = a\,\not{K} + b\,\not{u}$. To find out the coefficients we can calculate $Tr(\not{K}Re\Sigma)/4$ and $\not{T}r(\not{u}Re\Sigma)/4$. Using $Tr(\not{a}\,\not{b}) = 4a_\mu b^\mu = 4a \cdot b$

$$\begin{aligned}
&\frac{1}{4}Tr(\not{K}Re\Sigma) \qquad (4.20)\\
&= 2|f|^2 \int \frac{d^3p}{(2\pi)^3}\frac{1}{2p}\left(n_b(p)\frac{K\cdot K + K\cdot P^+}{(K+P^+)^2} + n_b(p)\frac{K\cdot K + K\cdot P^-}{(K+P^-)^2}\right)\\
&+ 2|f|^2 \int \frac{d^3p}{(2\pi)^3}\frac{1}{2p}\left(n_f(p)\frac{K\cdot P^+}{(K+P^+)^2} + n_f(p)\frac{K\cdot P^-}{(K+P^-)^2}\right)\\
&= \frac{|f|^2}{4\pi^2}\int_0^\infty p^2 dp \int_{-1}^1 dc \frac{1}{p}\\
&\times \left[n_b(p)\frac{K^2 + K_0 p - kpc}{K^2 + 2K_0 p - 2kpc} + n_b(p)\frac{K^2 - K_0 p - kpc}{K^2 - 2K_0 p - 2kpc}\right.\\
&\left. + n_f(p)\frac{K_0 p - kpc}{K^2 + 2K_0 p - 2kpc} + n_f(p)\frac{-K_0 p - kpc}{K^2 - 2K_0 p - 2kpc}\right]
\end{aligned}$$

Here we have written $c = \cos(\theta)$, where θ is the angle between $\mathbf{k}$ and $\mathbf{p}$. One should also note that $P^2 = P_\mu P^\mu = 0$. The angular integral is easily calculated. The terms are of form

$$I = \int_{-1}^1 \frac{A - Bc}{C - 2Bc} \tag{4.21}$$

Taking $c' = C - 2Bc$ we have

$$
\begin{aligned}
I &= \frac{1}{2B}\int_{C-2B}^{C+2B}\frac{A-c+c'}{2c'} = \frac{-1}{2B}\int_{C+2B}^{C-2B}\left(\frac{1}{2}+\frac{2A-C}{2c'}\right)\\
&= 1 - \frac{2A-C}{4B}\ln\left(\frac{C-2B}{C+2B}\right)
\end{aligned}
$$

Substituting A,B and C by the terms in 4.20 we have

$$
\begin{aligned}
&\frac{1}{4}Tr(\not{K}Re\Sigma)\\
&= \frac{|f|^2}{4\pi^2}\int_0^\infty dpn_b(p)\left(p - \frac{K^2}{4k}\log\frac{K^2+2K_0p-2kp}{K^2+2K_0p+2kp}\right.\\
&\left.+p - \frac{K^2}{4k}\log\frac{K^2-2K_0p-2kp}{K^2-2K_0p+2kp}\right)\\
&+ \frac{|f|^2}{4\pi^2}\int_0^\infty dpn_f(p)\left(p + \frac{K^2}{4k}\log\frac{K^2+2K_0p-2kp}{K^2+2K_0p+2kp}\right.\\
&\left.+ p + \frac{K^2}{4k}\log\frac{K^2-2K_0p-2kp}{K^2-2K_0p+2kp}\right)\\
&= \frac{|f|^2}{4\pi^2}\int_0^\infty dp\left(n_b(2p - \frac{K^2}{4k}L_1(p)) + n_f(p)(2p + \frac{K^2}{4k}L_1(p))\right),\\
&\text{where } L_1(p) = \log\left(\frac{(2p+K_0+k)(2p-K_0+k)}{(2p+K_0-k)(2p-K_0-k)}\right)
\end{aligned}
$$

Without the distribution functions the integral would diverge as p^2. However, roughly speaking, the distribution cuts the integral when $p \approx T$, giving the T^2 behavior. The logarithm L_1 diverges as $\log p$ giving a contribution relative to $\log T$ and we will omit it. Now the integral becomes

$$
\begin{aligned}
\frac{1}{4}Tr(\not{K}Re\Sigma) &= \frac{|f|^2}{4\pi^2}\int_0^\infty dp\,(2pn_b + 2pn_f(p)) \\
&= \frac{|f|^2}{4\pi^2}\left(\frac{\pi^2T^2}{6}+\frac{\pi^2T^2}{3}\right) = \frac{|f|^2T^2}{8}
\end{aligned}
\tag{4.22}
$$

This is, as expected, relative to T^2.

Next we should calculate $Tr(\not{u}Re\Sigma)/4$ to get both coeffiecients. This goes somewhat similarly, using $u = (1,0,0,0)$.

$$
\begin{aligned}
&\frac{1}{4}Tr(\not{u}Re\Sigma)\\
&\quad = 2|f|^2 \int \frac{d^3p}{(2\pi)^3}\frac{1}{2p}\left(n_b(p)\frac{u\cdot K + u\cdot P^+}{(K+P^+)^2} + n_b(p)\frac{u\cdot K + u\cdot P^-}{(K+P^-)^2}\right)\\
&\quad + 2|f|^2 \int \frac{d^3p}{(2\pi)^3}\frac{1}{2p}\left(n_f(p)\frac{u\cdot P^+}{(K+P^+)^2} + n_f(p)\frac{u\cdot P^-}{(K+P^-)^2}\right)\\
&\quad = \frac{|f|^2}{4\pi^2}\int_0^\infty pdp\int_{-1}^1 dc\frac{1}{p}\\
&\quad \times\left[n_b(p)\frac{K_0+p}{K^2+2K_0p-2kpc} + n_b(p)\frac{K_0-p}{K^2-2K_0p-2kpc}\right.\\
&\quad \left.+n_f(p)\frac{p}{K^2+2K_0p-2kpc} + n_f(p)\frac{-p}{K^2-2K_0p-2kpc}\right]\\
&\quad = \frac{|f|^2}{4\pi^2k}\int_0^\infty dpn_b(p)\\
&\quad \times\left(K_0L_1(p) + p\log\frac{K^2+2pK_0+2pk}{K^2+2pK_0-2pk} - p\log\frac{K^2-2pK_0+2pk}{K^2-2pK_0-2pk}\right)\\
&\quad + \frac{|f|^2}{4\pi^2k}\int_0^\infty dpn_f(p)\left(p\log\frac{K^2+2pK_0+2pk}{K^2+2pK_0-2pk} - p\log\frac{K^2-2pK_0+2pk}{K^2-2pK_0-2pk}\right)\\
&\quad = \frac{|f|^2}{4\pi^2k}\int_0^\infty dpn_b(p)\left(K_0L_1(p) + 2p\log\frac{K_0+k}{K_0-k} - pL_2(p)\right)\\
&\quad + \frac{|f|^2}{4\pi^2k}\int_0^\infty dpn_f(p)\left(2p\log\frac{K_0+k}{K_0-k} - pL_2(p)\right)\\
&\quad \text{where } L_2(p) = \log\left(\frac{(2p+K_0+k)(2p-K_0+k)}{(2p+K_0-k)(2p-K_0-k)}\right)
\end{aligned}
$$

The logarithm L_1 again diverges as $\ln p$ and can be ignored. L_2 converges as p^{-1}, so we will ignore it as well. We have

$$
\begin{aligned}
&\frac{1}{4}Tr(\not{u}Re\Sigma)\\
&\quad = \frac{|f|^2}{4\pi^2k}\int_0^\infty dp\left(n_b(p)2p\log\frac{K_0+k}{K_0-k} + n_f(p)2p\log\frac{K_0+k}{K_0-k}\right)\\
&\quad = \frac{|f|^2}{4\pi^2k}\left(\frac{\pi^2T^2}{6} + \frac{\pi^2T^2}{3}\right)\log\frac{K_0+k}{K_0-k} = \frac{|f|^2T^2}{16k}\log\frac{K_0+k}{K_0-k}
\end{aligned}
\tag{4.23}
$$

We assumed that $Re\Sigma = a\not{K} + b\not{u}$. This implies that $Tr(\not{u}Re\Sigma)/4 = au\cdot K + bu\cdot u = aK_0 + b$ and $Tr(\not{K}Re\Sigma) = aK\cdot K + bK\cdot u = aK^2 + bK_0$.

Using these, 4.22 and 4.23 we get

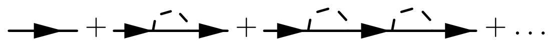
$$
\begin{aligned}
aK_0 + b &= \frac{|f|^2T^2}{16k}\log\frac{K_0+k}{K_0-k} \\
aK^2 + bK_0 &= \frac{|f|^2T^2}{8}
\end{aligned}
$$

Solving these we get

$$
\begin{aligned}
Re\Sigma &= a\not{K} + b\not{u} && (4.24)\\
&= -\frac{|f|^2T^2}{8k^2}\left(1-\frac{K_0}{2k}\log\frac{K_0+k}{K_0-k}\right)\not{K} && (4.25)\\
&+\frac{|f|^2T^2}{8k}\left(\frac{K_0}{k}-\frac{1}{2}\left(\frac{K_0^2}{k^2}-1\right)\log\frac{K_0+k}{K_0-k}\right)\not{u}
\end{aligned}
$$

The imaginary part of the diagram (equation 4.19) has two delta functions. This means that it only depends on k or K_0, even for large T, and we can ignore it.

This gives as a real self-energy that has the same form as mass in equation 2.7. The particle interacts while it propagates, but does not change. We can think of this as a mass in the same way: the progatator is

$G =$  (4.26)

$$
\begin{aligned}
&= S + S\Sigma S + S\Sigma S\Sigma S + \cdots = \frac{S}{1-\Sigma S}\\
&= \frac{1}{\gamma_\mu[(1-a)K) - bu]^\mu} = \frac{(1-a)\not{K} - b\not{u}}{[(1-a)K+bu]^2}.
\end{aligned}
$$

The normal free particle propagator has poles when $K = \pm m$ and for a massles particle they become $K = 0$. Our propagator for a particle in a plasma has poles, but they do not appear at $K = 0$. We may interpret these poles as the absolute value of a thermal mass the particle has due to its interactions with the surrounding plasma. We may solve them from

$$
\begin{aligned}
&[(1-a)K + bu]^2 = 0\\
\Rightarrow &(1-a)^2k^2 = [(1-a)K_0 - b)^2
\end{aligned}
$$

Taking the positive root we have

$$
\begin{aligned}
(1-a)k &= (1-a)K_0 - b && (4.27)\\
\Rightarrow K_0 - k &= \frac{M^2}{k}\left(1+\frac{1}{2}(1-\frac{K_0}{k})\log(\frac{K_0+k}{K_0-k})\right),
\end{aligned}
$$

where we have written $M^2 = f^2T^2/8$. One can numerically find the root of this equation using the variables k/M and K_0/M. At $k = 0$ we get

$K_0/M = 1$, so that the energy of a particle at rest is $K_0 = M$. We see that M really behaves as mass. 4.27 is the dispersion relation for a fermion in a thermal plasma, giving the energy of the particle. It is not of the same form as the free particle dispersion relation, $K_0 = \sqrt{k^2 + m^2}$, but behaves differently for large k. For a free particle we would then get $K_0 \approx k + m^2/2k$, but for particle in a plasma we find

$$K_0 \approx k + \frac{M^2}{k}. \tag{4.28}$$

We will use this energy as the effective hamiltonian of the particle later.

Diagrams b) and c) in figure 4.2 can be evaluated in exactly the same way. The differences are that there is a $-g_{\mu\nu}$ in the boson propagator and $i\gamma$:s at the vertices, giving a coefficient of 2, in b) there are three generators and the coupling is $g/2$ giving a total factor $3g^2/2$ and in c) the coupling is $g'/2$ with only one generator, giving $g'^2/2$. The total thermal mass squared of a left-handed quark in a high temperature plasma ($T > T_{\mathrm{EW}}$) is then

$$M^2 = T^2 \left(\frac{g'^2 + 3g^2}{32} + \frac{|f|^2}{8} \right) \tag{4.29}$$

It is worth noting that all other particles have thermal masses, too. For the right-handed, or sterile, neutrino it is created only trough the Higgs interaction, and as the couplings h_i are small, we may ignore it. The Higgs boson, however, has a larger mass. It was calculated to second order in [9] and with the Higgs mass $m = 129\mathrm{GeV}$ and expectation value $v = 246\mathrm{GeV}$ it becomes roughly $m_T^2 \approx 0.5T^2$. This is large enough to change the numerical value of both the previous and the following calculation, but not the magnitude. As we are only doing an order of magnitude analysis, we may use the zero-temperature mass for the Higgs boson.

4.2 Destruction rates for neutrinos

The possible destruction and creation of neutrinos of course plays an important role in the formation of the baryon asymmetry. We will now calculate the reaction rates associated with the first order destruction rates of neutrinos. The left- and right-handed neutrinos could decay into each other emitting a Higgs doublet. The rate of this decay would be suppressed by the effective Higgs mass. It is actually proportional to the imaginary part of the self-energy diagram in section 4.1, where we do neglect it. The leading order destruction channels are the two to two scatterings of a neutrino and the lepton doublet to a top quark and the top doublet, mediated by the Higgs particle. Figure 4.3 corresponds to the most important scatterings of neutrinos. Note that similar scatterings are be possible with electrons

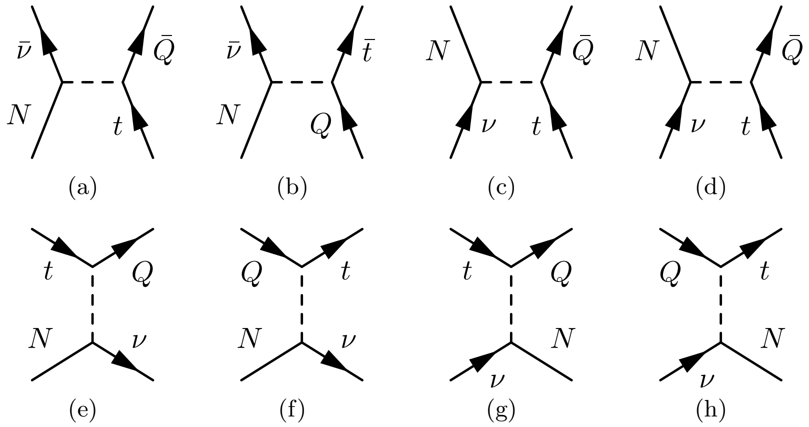


Figure 4.3: Highest order two to two decay channels of the left- and right-handed neutrinos. t is the right-handed top quark and Q can be the left-handed top- or bottom-quark.

instead of left-handed neutrinos. However, we assume that all particles, except neutrinos, are at thermal equilibrium, meaning that the process is as likely as the reversed one and there is no net change. Any excess electrons or positrons that are created will scatter back into neutrinos. Therefore we only need to consider the oscillations of neutrinos. Again, taking these rates into account would change the numerical value of calculation, but not the order of magnitude.

There are two basic types of diagrams here, an s- and a t-channel. We will calculate one example of both and the rest will have the same values. Four-momenta will be marked with capital letters, such as P_N, three-momenta with bold letters, $\mathbf{p}_N$, and the length of the three momenta with normal letters, p_N. The corresponding energy will be written as in P_{N0}, the zero component of the four-momentum. We are discussing temperatures high above the electroweak symmetry breaking point, so all particles are massless except for N, which still has the mass M, and the Higgs particle, that has the mass m. Thus for all other particles $P_0 = p$.

Let us start by calculating the rate corresponding to diagrams (e) and (f) in figure 4.3. We can do this using the normal methods of particle physics, but taking into account the distribution of particles in the plasma. The complete rate of the interactions in the diagram is given by the Feynman amplitude squared integrated over all the momenta

$$\begin{aligned}\Gamma &= \int \frac{d^3\mathbf{p_N}}{(2\pi)^3}\frac{n_f(p_N)}{2P_{N0}}\int \frac{d^3\mathbf{p}_t}{(2\pi)^3}\frac{n_f(p_t)}{2P_{t0}}\sigma \\ \sigma &= \int \frac{d^3\mathbf{p}_\nu}{(2\pi)^3}\frac{1}{2P_{\nu 0}}\int \frac{d^3\mathbf{p}_Q}{(2\pi)^3}\frac{1}{2P_{Q0}} \\ &\quad\times (2\pi)^4\delta^{(4)}(P_N + P_t - P_\nu - P_Q)|M|^2\end{aligned} \tag{4.30}$$

Here σ is the reduced cross-section and $|M|^2$ is the Feynman amplitude squared, summed over all final states and averaged over initial spins. In the integrals over the outgoing momenta we should in principle have the Pauli exclusion factor $1-n_f(p)$. In an order of magnitude analysis, where we keep only the leading order terms, we can ignore it, just replacing it with 1. The right-handed neutrinos aren't actually in thermal equilibrium and we want to calculate the reaction rate of a particle of given momentum, k. To this end we take the derivative with respect to p_N and divide the distribution function out to normalize the distribution to unity. $d\Omega$ is the integral over the direction of the momentum.

$$\Gamma(p_N) = \frac{\int d\Omega}{(2\pi)^3 2P_{N0}} \int \frac{d^3\mathbf{p}_t}{(2\pi)^3} \frac{n_f(p_t)}{2P_{t0}} \sigma(P_\nu, P_Q) \tag{4.31}$$

The distribution functions are are defined in the rest-frame of the surrounding plasma, so the second integral has to be performed in it. As the cross-section is Lorentz invariant, we can choose to calculate it in the center of mass frame, with $\mathbf{p}_N = \mathbf{p} = -\mathbf{p}_t$. Taking the integral over $\mathbf{p}_Q$ using the delta-function we get $\mathbf{p}_\nu = \mathbf{q} = -\mathbf{p}_Q$. Thus

$$\begin{aligned}\sigma = &\int \frac{d^3\mathbf{q}}{(2\pi)^3} \frac{1}{4q^2} \\ &\times (2\pi)\delta(\sqrt{p^2+M^2}+p-2q)|M|^2\end{aligned} \tag{4.32}$$

The amplitude is now

$$M = it\bar{Q}\frac{2fh}{P_\psi^2 - m^2} l\bar{N}, \tag{4.33}$$

where $P_\psi = P_t - P_Q = (p-q, \mathbf{q}-\mathbf{p})$, f is the Yukawa coupling of neutrinos and h that of the top quark. We will use a transformation $c = \cos(\theta)$, where theta is the angle between $\mathbf{p}$ and $\mathbf{q}$. This way the three dimensional integral becomes $\int d^3\mathbf{q} = \int dq \int_{-1}^{1} dc \int_0^{2\pi} d\phi$. Averaging over initial spins and summing over final states we get

$$\begin{aligned}|M|^2 &= 2 \times 16 \frac{f^2h^2}{(P_\psi^2 - m^2)^2} P_N \cdot P_e P_t \cdot P_Q \\ &= 32 \frac{f^2h^2}{(-2pq(1-c)-m^2)^2}(q\sqrt{p^2+M^2} - pqc)pq(1-c)\end{aligned} \tag{4.34}$$

The first coefficient 2 comes from the two possible particles in Q. Combining 4.34 and 4.32

$$\sigma = \frac{8f^2h^2}{2\pi}\int dq \int_{-1}^{1} dc\delta(\sqrt{p^2+M^2}+p-2q) \tag{4.35}$$

$$\times \frac{(q\sqrt{p^2+M^2}-pqc)pq(1-c)}{(-2pq(1-c)-m^2)^2} \tag{4.36}$$

To simplify the integral over c we write

$$c' = c - \frac{m^2}{2pq} - 1 \Rightarrow \tag{4.37}$$

$$\sigma = \frac{8f^2h^2}{2\pi}\int dq\delta(\sqrt{p^2+M^2}+p-2q)$$

$$\times \int_{-\frac{m^2}{2pq}-2}^{-\frac{m^2}{2pq}} dc' \frac{(q\sqrt{p^2+M^2}-pqc'-pq-\frac{m^2}{2})(c'-\frac{m^2}{2})}{4p^2q^2(c')^2}$$

$$= \frac{8f^2h^2}{2\pi}\int dq\delta(\sqrt{p^2+M^2}+p-2q)\int_{-\frac{m^2}{2pq}-2}^{-\frac{m^2}{2pq}} dc'$$

$$\times \left[\frac{1}{4}+\frac{m^2+pq-q\sqrt{p^2+M^2}}{4pqc'}+\frac{m^2(m^2+2pq-2q\sqrt{p^2+M^2})}{16p^2q^2(c')^2}\right]$$

$$= \frac{8f^2h^2}{2\pi}\int dq\frac{\delta(q-(\sqrt{(p^2+M^2)}+p))}{2}\left[\frac{m^2+3pq-q\sqrt{p^2+M^2}}{m^2+4pq}\right.$$

$$\left.+\frac{m^2+pq-q\sqrt{p^2+M^2}}{4pq}\ln\left(\frac{m^2}{m^2+4pq}\right)\right]$$

The integration over q gives $q = (\sqrt{p^2+M^2}+p)/2$. Applying this and doing the necessary algebra the cross-section becomes

$$\sigma = \frac{4f^2h^2}{(2\pi)}\left[\frac{2m^2+2p^2+2p\sqrt{p^2+M^2}-M^2}{2m^2+4p^2+4p\sqrt{p^2+M^2}}\right. \tag{4.38}$$

$$\left.+\frac{2m^2+M^2}{4p^2+4p\sqrt{p^2+M^2}+4M^2}\ln\left(\frac{m^2}{m^2+2p^2+2p\sqrt{p^2+M^2}}\right)\right]$$

We need to use Lorentz-invariant variables, so we write this using the Mandlestam variable $s = (P_N+P_t)^2 = 2p^2+2p\sqrt{p^2+M^2}+M^2$. The following then holds for all coordinate-systems:

$$\sigma = \frac{4f^2h^2}{(2\pi)}\left[\frac{2m^2-2M^2+s}{2m^2-2M^2+2s}+\frac{2m^2+M^2}{s+2M^2}\ln\left(\frac{m^2}{m^2+s-M^2}\right)\right] \tag{4.39}$$

We must work in a frame moving along with the plasma. In this system the momentum of the neutrino is $\mathbf{k}$ with $k \sim T$. As the velocity of the plasma is zero, we are free to choose the direction of the coordinatesystem. The rate cannot depend on the Lorentz frame we choose and therefore can only depend on the lengths of the momenta and the angle between them. It can not depend on the absolute direction of $\mathbf{p}_N$. This means that we can take the first angle integral in 4.31 independently giving just 4π. Then we can take the z-axis to be to the direction of $\mathbf{p}_N$. In these coordinates $s = 2kp_t(1-c_2)+M^2$, where $c_2 = \cos(\theta_{Nt})$ is the cosine of the angle between $\mathbf{p}_N$ and $\mathbf{p}_t$. The destruction rate associated with the diagram is then

$$\Gamma(k) = \frac{1}{k}\int \frac{d^3\mathbf{p}_t}{(2\pi)^5}\frac{n_f(p_t)}{2p_t}\frac{4f^2h^2}{(2\pi)}\left[\frac{2m^2 - M^2 + 2kp_t(1-c_2)}{2m^2 + 4kp_t(1-c_2)} + \frac{2m^2+M^2}{2kp_t(1-c_2)+3M^2}\ln\left(\frac{m^2}{m^2+2kp_t(1-c_2)}\right)\right] \tag{4.40}$$

We will then calculate the angle integral over c_2. It is

$$I = \int_{-1}^{1} dc_2 \left[\frac{2m^2 - M^2 + 2kp_t(1-c_2)}{2m^2 + 4kp_t(1-c_2)} + \frac{2m^2+M^2}{2kp_t(1-c_2)+3M^2}\ln\left(\frac{m^2}{m^2+2kp_t(1-c_2)}\right)\right] \tag{4.41}$$

We may simplify the first term by taking $c_2' = 2m^2 + 4kp_t(1-c_2)$ to have

$$\begin{aligned} I_1 &= \frac{1}{4kp_t}\int_{2m^2}^{2m^2+8kp_t} dc_2' \left(\frac{1}{2} + \frac{m^2 - M^2}{c_2'}\right) \\ &= 1 + \frac{m^2-M^2}{8kp_t}\ln\left(\frac{2m^2+8kp_t}{2m^2}\right) \end{aligned} \tag{4.42}$$

The second term is slightly more complicated and we will take $c_2' = 3M^2 + 2kp_t(1-c_2)$ giving

$$\begin{aligned} I_2 &= \frac{2m^2+M^2}{2kp_t}\int_{3M^2}^{3M^2+4kp_t} dc_2' \frac{1}{c_2'}\ln\left(\frac{m^2}{c_2' + m^2 - 3M^2}\right) \\ &= \frac{2m^2+M^2}{2kp_t}\int_{3M^2}^{3M^2+4kp_t} dc_2' \frac{\ln(m^2) - \ln(c_2' + m^2 - 3M^2)}{c_2'} \\ &= \frac{(2m^2+M^2)\ln(m^2)}{2kp_t}\ln\left(\frac{3M^2+4kp_t}{3M^2}\right) \\ &- \frac{2m^2+M^2}{2kp_t}\int_{3M^2}^{3M^2+4kp_t} dc_2' \frac{\ln(c_2' + m^2 - 3M^2)}{c_2'} \end{aligned} \tag{4.43}$$

Now writing

$$c_2' = -(m^2 - 3M^2)t \tag{4.44}$$

we have

$$\begin{aligned}
I_2 &= \frac{(2m^2+M^2)\ln(m^2)}{2kp_t}\ln\left(\frac{3M^2+4kp_t}{3M^2}\right) \\
&- \frac{2m^2+M^2}{2kp_t}\int_{-\frac{3M^2}{m^2-3M^2}}^{-\frac{3M^2+4kp_t}{m^2-3M^2}} dt\frac{\ln(m^2-3M^2)+\ln(1-t)}{t} \\
&= \frac{(2m^2+M^2)\ln(m^2)}{2kp_t}\ln\left(\frac{3M^2+4kp_t}{3M^2}\right) \\
&- \frac{(2m^2+M^2)\ln(m^2-3M^2)}{2kp_t}\ln\left(\frac{3M^2+4kp_t}{3M^2}\right) \\
&- \frac{2m^2+M^2}{2kp_t}\int_{-\frac{3M^2}{m^2-3M^2}}^{-\frac{3M^2+4kp_t}{m^2-3M^2}} dt\frac{\ln(1-t)}{t} \\
&= \frac{(2m^2+M^2)}{2kp_t}\ln\left(\frac{m^2}{m^2-3M^3}\right)\ln\left(\frac{3M^2+4kp_t}{3M^2}\right) \\
&- \frac{2m^2+M^2}{2kp_t}\left(\mathrm{Li}_2\left[-\frac{3M^2+4kp_t}{m^2-3M^2}\right] - \mathrm{Li}_2\left[-\frac{3M^2}{m^2-3M^2}\right]\right)
\end{aligned} \tag{4.45}$$

Here Li_2 is the dilogarithm. It has many known properties some of which can be found in [20]. We have used its integral representation

$$\mathrm{Li}_2(z) = \int_0^z dt\frac{\ln(1-t)}{t}. \tag{4.46}$$

Combining 4.42 and 4.45 we have

$$\begin{aligned}
I = I_1 + I_2 &= 1 + \frac{m^2-M^2}{8kp_t}\ln\left(\frac{2m^2+8kp_t}{2m^2}\right) \\
&+ \frac{(2m^2+M^2)}{2kp_t}\ln\left(\frac{m^2}{m^2-3M^3}\right)\ln\left(\frac{3M^2+4kp_t}{3M^2}\right) \\
&+ \frac{2m^2+M^2}{2kp_t}\left(\mathrm{Li}_2\left[-\frac{3M^2}{m^2-3M^2}\right] - \mathrm{Li}_2\left[-\frac{3M^2+4kp_t}{m^2-3M^2}\right]\right)
\end{aligned} \tag{4.47}$$

Coming back to the calculation of the rate, we have from 4.40

$$\Gamma(k) = \frac{2f^2h^2}{T}\int\frac{dp_t}{(2\pi)^5}p_t n_f(p_t) I \tag{4.48}$$

Roughly speaking the distribution function cuts the integral around $p_t \sim T$. If the integral were to diverge as p_t^2, this would give a T^2 behavior. The first term in I gives the fastest divergence, and therefore the highest order in T, and we can neglect the other terms and use $I = 1$

$$\Gamma(k) = \frac{2f^2h^2}{T}\int \frac{dp_t}{(2\pi)^5} p_t n_f(p_t) \tag{4.49}$$

The integral over the distribution function is

$$\int dp n_f(p) p = \frac{\pi^2 T^2}{12}. \tag{4.50}$$

$$\Gamma(k) = \frac{f^2h^2}{(2\pi)^5 T}\frac{\pi^2T^2}{6} = \frac{f^2h^2}{16\pi^3}\frac{T}{12} \tag{4.51}$$

We still need to take into account that there are three possible colors of the quarks, two different diagrams and the same two diagrams with antiquarks instead of quarks, altogether giving a factor of 12. The value of h is roughly 1, giving

$$\Gamma(k) = \frac{f^2 T}{16\pi^3} \tag{4.52}$$

The rate corresponding to the diagrams (a) and (b) is calculated similarly. In the center of mass frame $\mathbf{p}_N = \mathbf{p} = -\mathbf{p}_\nu$ and taking the integral with the delta function we have $\mathbf{p}_t = \mathbf{q} = -\mathbf{p}_Q$. Using these $|M|^2$ has the same form (4.34), except that $P_\psi = P_t + P_Q = (2q, \mathbf{0})$. Here again we use $c = \cos(\theta)$, where θ is the angle between $\mathbf{p}$ and $\mathbf{q}$. These give

$$\begin{aligned}
\sigma &= \frac{8f^2h^2}{2\pi}\int dq \int_{-1}^{1} dc\delta(\sqrt{p^2+M^2}+p-2q)\frac{(q\sqrt{p^2+M^2}-pqc)pq(1-c)}{(4q^2-m^2)^2} \\
&= \frac{8f^2h^2}{2\pi}\int dq\delta(\sqrt{p^2+M^2}+p-2q) \\
&\times \int_{-1}^{1} dc\frac{pq(q\sqrt{p^2+M^2}-(q\sqrt{p^2+M^2}+pq)c+pqc^2)}{(4q^2-m^2)^2} \\
&= \frac{8f^2h^2}{2\pi}\int dq\frac{\delta(q-(\sqrt{(p^2+M^2)}+p))}{2}\frac{2pq(pq+3q\sqrt{p^2+M^2})}{3(4q^2-m^2)^2}
\end{aligned}$$

Using $q = (p + p\sqrt{p^2+M^2})/2$ and $s = 2p^2 + 2p\sqrt{p^2+M^2} + M^2$ we have

$$\begin{aligned}
\sigma &= \frac{4f^2h^2}{2\pi}\frac{(p^2+p\sqrt{p^2+M^2})(2p^2+2p\sqrt{p^2+M^2}+\frac{3}{2}M^2)}{3(2p^2+2p\sqrt{p^2+M^2}+M^2-m^2)^2} \\
&= \frac{2f^2h^2}{2\pi}\frac{(s-M^2)(s+\frac{1}{2}M^2)}{3(s-m^2)^2}
\end{aligned}$$

Going to the rest-frame of the plasma, with $s = 2kp_e(1-c_2)+M^2$, where c_2 is the cosine of the angle between $\mathbf{p}_N$ and $\mathbf{p}_\nu$, we have

$$\Gamma(k) = \frac{1}{k}\int \frac{d^3\mathbf{p}_\nu}{(2\pi)^5}\frac{n_f(p_\nu)}{2p_\nu}\frac{2f^2h^2}{2\pi}\frac{(s-M^2)(s+\frac{1}{2}M^2)}{3(s-m^2)^2} \tag{4.53}$$

$$= \frac{1}{k}\int \frac{d^3\mathbf{p}_\nu}{(2\pi)^5}\frac{n_f(p_\nu)}{2p_\nu}\frac{2f^2h^2}{2\pi}\frac{(2kp_\nu(1-c_2))(2kp_\nu(1-c_2)+\frac{3}{2}M^2)}{3(2kp_\nu(1-c_2)+M^2-m^2)^2} \tag{4.54}$$

We may simplify this integral by taking $c_2' = 2kp_\nu(1-c_2)+M^2-m^2$

$$\Gamma(k) = \frac{f^2h^2}{6k^2}\int \frac{dp_\nu n_f(p_\nu)}{(2\pi)^5}\int_{M^2+m^2}^{4kp_\nu+M^2+m^2} dc_2' \frac{(c_2'-M^2+m^2)(c_2'+\frac{1}{2}M^2+m^2)}{(c_2')^2}$$

$$= \frac{f^2h^2}{6k^2}\int \frac{dp_\nu n_f(p_\nu)}{(2\pi)^5}\int dc_2' \frac{(c_2')^2+(2m^2-\frac{1}{2}M^2)c_2'+(m^4-\frac{1}{2}M^4-\frac{1}{2}m^2M^2)}{(c_2')^2}$$

$$= \frac{f^2h^2}{6k^2}\int \frac{dp_\nu n_f(p_\nu)}{(2\pi)^5}\left[4kp_\nu + \frac{4m^2-M^2}{2}\ln\left(\frac{4kp_\nu+M^2+m^2}{M^2+m^2}\right)\right.$$

$$\left. + \frac{2m^4-M^4-m^2M^2}{2}\left(\frac{1}{M^2+m^2}-\frac{1}{4kp_\nu+M^2+m^2}\right)\right]$$

$$= \frac{f^2h^2}{6k}\int \frac{dp_\nu n_f(p_\nu)p_\nu}{(2\pi)^5}\left[4 + \frac{4m^2-M^2}{2kp_\nu}\ln\left(\frac{4kp_\nu+M^2+m^2}{M^2+m^2}\right)\right.$$

$$\left. + 2\frac{2m^4-M^4-m^2M^2}{(M^2+m^2)(4kp_\nu+M^2+m^2)}\right]$$

Again, the distribution function cuts the integral and the highest order in T is given by the first term, which is the one that diverges fastest. This gives

$$\Gamma(k) = \frac{2f^2h^2}{3k}\int \frac{dp_\nu}{(2\pi)^5}p_\nu n_f(p_\nu) \tag{4.55}$$

$$= \frac{2f^2h^2}{3(2\pi)^5T}\frac{\pi^2T^2}{12} = \frac{1}{3}\frac{f^2h^2}{16\pi^3}\frac{T}{12} \tag{4.56}$$

There are again three possible quark colors and two diagrams and we can take $h \sim 1$, leaving us with

$$\Gamma = \frac{f^2T}{6\times 16\pi^3} \tag{4.57}$$

Adding up these two rates 4.52 and 4.57 we get the destruction rate of right-handed neutrinos in processes not involving left-handed neutrinos:

$$\Gamma = \frac{7f^2T}{96\pi^3} \tag{4.58}$$

As the diagrams are similar to the other possible scatterings and the Majorana mass doesn't contribute at the first order, the destruction rates associated with them are the same. These reaction include the ones that create a ν or destroy a $\bar{\nu}$ (diagram c,d,g and h) and similar reaction for left-handed neutrinos. These are now the most important diagrams of the neutrino oscillations and play a very important role in the evolution of the baryon asymmetry.

Chapter 5

Baryon asymmetry in the νMSM

5.1 Solving the baryon asymmetry

It is natural to assume that the big bang model is valid at temperatures far above the electro-weak scale. We will also assume that the initial concentration of right-handed neutrinos is zero. Their Yukawa couplings are small and so they would never equilibrate. At the very least the lightest of them should interact weakly enough never to equilibrate, as it plays the role of dark matter in our current Universe. Their concentration would then be preserved to the day.

We will calculate the baryon asymmetry produced following the methods in the articles of Asaka and Shaposhnikov in 2005 [7] and that of Akmedov, Rubakov and Smirnov in 1998 [10]. In their scheme the asymmetry is produced by right-handed neutrino oscillations with CP-violating phases. The total lepton number, the number of standard model leptons and total helicity of right-handed neutrinos, is conserved in these oscillations, but the lepton number in left-handed neutrinos becomes nonzero and is translated to baryon number trough sphaleron processes. The baryon number generated is of the same scale as the lepton asymmetry when the sphalerons stop, as calculated in section 3.2 and in [12].

Neutrinos are out of thermal equilibrium, and so we have to describe their time evolution using their density matrix. This is a 12×12 matrix and a function of temperature and the momentum of the particle. Each diagonal entry gives the density of a type of particle and each non-diagonal entry gives the probability of the first type turning into the other. The values are normalized so that the diagonal components are the occupation number of a state with the given momentum. This way we can describe both the effective quantum effects and the thermal effects. Here the entries are themselves 3×3 matrices describing the three generations. We will use the indices L and $\bar{L}$ for active neutrinos ν_L and their antineutrinos ν_L^C respectively, and N and $\bar{N}$ for the different helicity states of the sterile neutrino. We may write the density matrix as

$$\rho = \begin{pmatrix} \rho_{LL} & \rho_{L\bar{L}} & \rho_{LN} & \rho_{L\bar{N}} \\ \rho_{\bar{L}L} & \rho_{\bar{L}\bar{L}} & \rho_{\bar{L}N} & \rho_{\bar{L}\bar{N}} \\ \rho_{NL} & \rho_{N\bar{L}} & \rho_{NN} & \rho_{N\bar{N}} \\ \rho_{\bar{N}L} & \rho_{\bar{N}\bar{L}} & \rho_{\bar{N}N} & \rho_{\bar{N}\bar{N}} \end{pmatrix} \tag{5.1}$$

It satisfies the evolution equation [23]

$$i\frac{d\rho}{dt} = [H, \rho] - \frac{i}{2}\{\Gamma, \rho\} + \frac{i}{2}\{\Gamma^p, 1 - \rho\}, \tag{5.2}$$

where $H = k(t) + H^0 + H^{int}$ is the effective hamiltonian describing the medium effects on neutrino propagation and $k(t) \sim T$ is the momentum of neutrinos. Γ is the destruction rate and Γ^p the production rate.

There is a Pauli exclusion factor $1 - \rho$ in the last term. We can neglect it in an order of magnitude analysis and replace the last term with $i\Gamma^p$. If now the system were at thermal equilibrium,

$$\rho = \rho^{eq} = e^{-H/T}, \tag{5.3}$$

5.2 would give

$$\Gamma^p = \frac{1}{2}\{\Gamma, \rho^{eq}\} \tag{5.4}$$

Using this we may write the equilibrium part of the production rate in the destruction rate. What is left is the production rate given by the departure of equilibrium of the other particle.

$$\frac{d\rho}{dt} = -i[H, \rho] - \frac{1}{2}\{\Gamma, \rho - \rho^{eq}\} + (\Gamma^p - \Gamma^p_{eq}) \tag{5.5}$$

The last term is nonzero only for the processes involving particles that are out of equilibrium. In the leading order approximation for the constant part of the effective hamiltonian, H^0, we neglect the Yukawa couplings h_{ij}. H^0 becomes diagonal, with the mass part of the energy as the diagonal entry. For right-handed neutrinos this is just $M^2/2k(t)$ from the normal dispersion relation $H = \sqrt{k^2 + M^2}$. We have already calculated the effective mass of the left-handed neutrinos and from the dispersion relation 4.28 and the mass 4.29 we have

$$H^0_{LL} = H^0_{\bar{L}\bar{L}} = \frac{T^2}{k(t)}\left[\frac{g'^2 + 3q^2}{32}\text{diag}(1,1,1) + \frac{1}{8}\text{diag}(f_1^2, f_2^2, f_3^2)\right] \tag{5.6}$$

$$H^0_{NN} = H^0_{\bar{N}\bar{N}} = \frac{1}{2k(t)}\text{diag}(M_1^2, M_2^2, M_3^2)$$

The time dependence introduced by H^0 can be removed by taking $U(t) = e^{-i\int_0^t dt' H^0(t')}$ and $\rho = U(t)\tilde{\rho}U^\dagger(t)$. This brings us to the interaction picture, as the only part of the hamiltonian left is H^{int} describing interactions between different kinds of neutrinos. As we will later see, the most important temperature scale for leptogenesis is $T \sim (M^2 \times M^0)^{1/3}$, where the $M^0 = 7 \cdot 10^{17} GeV$ comes from the relation of time and temperature

$$t = \frac{M^0}{2T^2} \tag{5.7}$$

This is much higher than any mass scale in the theory and we can therefore ignore many nondiagonal terms in the matrix. All terms mixing N or $\bar{N}$ with L or $\bar{L}$ are suppressed because of the great difference between their effective masses. The differences between the masses of different generations of neutrinos is also large, so we can neglect all nondiagonal terms in ρ_{LL} and $\rho_{\bar{L}\bar{L}}$. Terms mixing L with $\bar{L}$ violate lepton number directly and are suppressed by the small Yukawa couplings. Terms mixing N with $\bar{N}$ involve a helicity flip and are suppressed by the small ratio of mass to temperature. Mathematically speaking, for all these matrix elements U oscillates very rapidly, averaging to zero.

We are now left with all of the diagonal elements of the matrix ρ_{LL} and $\rho_{\bar{L}\bar{L}}$ and all elements of ρ_{NN} and $\rho_{\bar{N}\bar{N}}$. These describe the densities of active neutrinos and the densities and mixings of the sterile neutrinos. The kinetic equations now take the form

$$\begin{aligned}\frac{d\tilde{\rho}}{dt} &= -i[U^\dagger(t)H^{int}(t)U(t), \tilde{\rho}] \\ &- U^\dagger(t)\frac{1}{2}[\Gamma, \rho - \rho^{eq}]U(t) + U^\dagger(t)(\Gamma^p - \Gamma^p_{eq})U(t)\end{aligned} \tag{5.8}$$

H^{int} describes interactions in which neutrinos change generation. The main channel is a one loop diagram $N_i \to \Psi l \to N_j$ with Higgs- and lepton doublets in the middle. It is exactly similar to the one depicted in figure 4.2 a) and has the same effective value, given by the second term of 5.6. Absorbing the U-matrices in its definition and taking $k(t) \sim T$ we have

$$H_N^{int}(t) = \frac{T}{8}U^\dagger(t)K_R h_d^2 K_R^\dagger U(t) \tag{5.9}$$

For sterile neutrinos all the reaction in figure 4.3 are possible and the generation of the particle can change in the process. The rate of this reaction is the one in equation 4.58, if we replace f with the corresponding element of the mass matrix $G = K_L P_\alpha h_d K_R^\dagger P_\beta$ in equation 2.18. These correspond to the diagrams a), b), e) and f). Diagrams c), d), g) and h) describe the transfer of lepton number from active to sterile neutrinos and have the same rates. Active neutrinos have the similar terms, except that generation changing interactions are suppressed and the H^{int} part dissappears.

In the first process, not only sterile leptons are destroyed, but active lepton number is generated. The difference is accounted by the last term in 5.8. The production rate given by this effect has the same value (4.58), but we need to replace f^2 with the corresponding element of $\Gamma^p = G(\rho_{NN})G^\dagger$. The ρ matrices are in the middle to give the density corresponding to the correct generation.

The equations for $\tilde{\rho}_{NN}$ and ρ_{LL} are then

$$\frac{d\tilde{\rho}_{NN}}{dt} = -i[H_N^{int}(t), \tilde{\rho}_{NN}] + \frac{7T}{2\times 96\pi^3}\left\{U^\dagger(t)G^\dagger G U(t), (\tilde{\rho}_{NN} - \tilde{\rho}_{NN}^{eq})\right\} \tag{5.10}$$

$$- \frac{7T}{2\times 96\pi^3}U^\dagger(t)G^\dagger(\tilde{\rho}_{LL} - \tilde{\rho}_{LL}^{eq})GU(t)$$

$$\frac{d\rho_{LL}}{dt} = \mathrm{diag}\left(\frac{7T}{96\pi^3}\left\{U^\dagger(t)GG^\dagger U(t), (\rho_{LL} - \rho_{LL}^{eq})\right\}\right. \tag{5.11}$$

$$\left. -\frac{7T}{96\pi^3}GU(t)(\tilde{\rho}_{NN} - \tilde{\rho}_{NN}^{eq})U^\dagger(t)G^\dagger\right)$$

The equations for $\tilde{\rho}_{\bar{N}\bar{N}}$ and $\rho_{\bar{L}\bar{L}}$ are the same with CP conjugation. The second term is 5.11 describes the transfer of lepton number from sterile to active neutrinos. We see that the trace of the second term in 5.10 is equal and opposite to the trace of the second term of 5.11. This ensures that total lepton number is conserved even though the amount of active neutrinos changes. The first term in 5.11 describes the transfer of lepton number to the sterile neutrinos. Without it the system would never thermalize. Its counterpart in 5.10 is the third term and their traces are also equal and opposite, ensuring that no lepton number is lost. The first term in 5.10 describes transfer of lepton number between the generations of sterile neutrinos.

With sufficiently small Yukawa couplings $h_d^2 \sim 2\times 10^{-14}$ [7] we can solve the system perturbatively. Enough active neutrinos are created during reheating to make sure that they are at thermal equilibrium, giving $\rho_{LL} - \rho_{LL}^{eq} = 0$. Sterile neutrinos are created only trough neutrino oscillations, meaning that initially $\tilde{\rho}_{NN} = 0$. They are created due to the third term in 5.10 and CP is violated in the process. This is transferred to the active neutrinos due to the second term in 5.11. The process freezes at $T = (M^2 \times M^0)^{1/3}$. The small asymmetry generated then causes a larger asymmetry in the amount of sterile neutrinos and finally in active neutrinos.

Let us first handle the generation of asymmetries in active neutrino flavors produced by the out of equilibrium density of the sterile neutrinos. This is produced by the second term in 5.11 and in the leading order, with $\rho_{LL} - \rho_{LL}^{eq} = 0$, the first term becomes zero. For $\Delta_L = \rho_{LL} - \rho_{\bar{L}\bar{L}}$ we have

$$\begin{aligned}\frac{d\Delta_L}{dt} &= -\frac{7T}{96\pi^3}\mathrm{diag}\left(G(\rho_{NN}-\rho^{eq}_{NN})G^\dagger - (G(\rho_{\bar{N}\bar{N}}-\rho^{eq}_{\bar{N}\bar{N}})G^\dagger)^*\right) \\ &= -\frac{7T}{96\pi^3}\mathrm{diag}\left(G\rho_{NN}G^\dagger - (G\rho_{\bar{N}\bar{N}}G^\dagger)^*\right) \\ &+ \frac{7T}{96\pi^3}\mathrm{diag}\left(Ge^{H/T}G^\dagger - (Ge^{H/T}G^\dagger)^*\right)\end{aligned} \tag{5.12}$$

All of the different terms of the matrix multiplication in the last term are real, giving

$$\begin{aligned}\frac{d\Delta_L}{dt} &= -\frac{7T}{96\pi^3}\mathrm{diag}\left(G\rho_{NN}G^\dagger - (G\rho_{\bar{N}\bar{N}}G^\dagger)^*\right) \\ &= -\frac{7T}{96\pi^3}\mathrm{diag}\left(GU\tilde{\rho}_{NN}U^\dagger G^\dagger - (GU\tilde{\rho}_{\bar{N}\bar{N}}U^\dagger G^\dagger)^*\right)\end{aligned} \tag{5.13}$$

We see that the first order effect ($\rho_{NN} = \rho_{\bar{N}\bar{N}} = 0$) disappears. We get to the second order by using 5.10 in the first order, with $\rho_{NN} = 0$. Only the second term of 5.10 survives and using $\tilde{\rho}^{eq} = U^\dagger e^{-H/T}U$ we have

$$\begin{aligned}\tilde{\rho}_{NN} &= -\frac{7}{192\pi^3}\int_0^t dt' T(t')\{U^\dagger(t')G^\dagger G U(t'), \tilde{\rho}^{eq}_{NN}\} \\ &= -\frac{7}{192\pi^3}\int_0^t dt' T\left(U^\dagger G^\dagger G e^{-H/T}U + U^\dagger e^{-H/T}G^\dagger G U\right)\end{aligned} \tag{5.14}$$

Using the time temperature relation

$$t = \frac{M^0}{2T^2} \tag{5.15}$$

and the interaction free part of the effective hamiltonian

$$H = T + \frac{M^2}{2T}, \tag{5.16}$$

where M^2 is the diagonal matrix of Majorana masses $M = \mathrm{diag}(M_1^2, M_2^2, M_3^2)$, we have for the density at the electroweak scale

$$\begin{aligned}\tilde{\rho}_{NN} = -\frac{7}{192\pi^3}\sqrt{\frac{M^0}{2}}\int_0^t dt'(t')^{-1/2}&\left(U^\dagger G^\dagger G e^{-1-tM^2/M^0}U\right. \\ &\left.+U^\dagger e^{-1-tM^2/M^0}G^\dagger G U\right)\end{aligned} \tag{5.17}$$

We should here separate the elements of the matrix to avoid getting confused with the indices. We use I and J to label the elements. Note that we don't sum over the matrix indices even though they are repeated.

$$\tilde{\rho}_{NN,IJ} = -\frac{7}{192\pi^3}\sqrt{\frac{M^0}{2}}\int_0^t dt'(t')^{-1/2}\left(U_I^\dagger (G^\dagger G)_{IJ} e^{-1-tM_J^2/M^0} U_J \right. \\ \left. + U_I^\dagger e^{-1-tM_I^2/M^0}(G^\dagger G)_{IJ} U_J\right) \tag{5.18}$$

The $U(t')$ matrix is time-dependent, but it only oscillates with the absolute value 1. Writing $U_I^\dagger U_J$ out we have

$$U_I^\dagger(t')U_J(t') = e^{-i\frac{M_I^2 - M_J^2}{2}\sqrt{\frac{2}{M^0}}\int_0^{t'} dt"(t")^{1/2}} \\ = e^{-i\frac{\Delta M^2}{3}\sqrt{\frac{2}{M^0}}(t')^{3/2}}. \tag{5.19}$$

Here $\Delta M^2 = \left|M_I^2 - M_J^2\right|$. The exponent is of order one when $T \sim (\Delta M^2 M^0)^{1/3}$. This is the reason that this is the most important temperature scale for leptogenesis. In terms of time

$$t \sim \frac{3^{2/3}(M^0)^{1/3}}{2^{1/3}(\Delta M^2)^{2/3}} \tag{5.20}$$

At any later times $U_I^\dagger U_J$ oscillates rapidly and averages out to zero when taking the integral. It would be rather hard to calculate analytically, but in an order of magnitude analysis, it is sufficient cut the integral at

$$t' = \frac{3^{2/3}(M^0)^{1/3}}{2^{1/3}(\Delta M^2)^{2/3}} \tag{5.21}$$

and take U and $U^\dagger$ to be constant. For the terms $I = J$ the cut time of course is infinite, as there is no oscillation. We will later see that they produce no lepton number.

$$\tilde{\rho}_{NN,IJ} = -\frac{7}{e192\pi^3}\sqrt{\frac{M^0}{2}}\Theta[\frac{3^{2/3}(M^0)^{1/3}}{2^{1/3}(\Delta M^2)^{2/3}} - t]U_I^\dagger\left[\int_0^t dt'(t')^{-1/2} \right. \\ \left. \times\left((G^\dagger G)_{IJ} e^{-tM_J^2/M^0} + e^{-tM_I^2/M^0}(G^\dagger G)_{IJ}\right)\right] U_J. \tag{5.22}$$

Now writing $l = t'M_I^2/M^0$ or $t'M_J^2/M^0$ we can identify the integral, without the $G^\dagger G$ matrix, as the lower incomplete gamma function

$$\gamma(\frac{1}{2}, t\frac{M_I^2}{M^0}) = \int_0^{t\frac{M_I^2}{M^0}} dl l^{-1/2} e^{-l} \tag{5.23}$$

Then

$$\tilde{\rho}_{NN,IJ} = -\frac{7}{e192\pi^3}\frac{M^0}{\sqrt{2}}\Theta[\frac{3^{2/3}(M^0)^{1/3}}{2^{1/3}(\Delta M^2)^{2/3}} - t][U_I^\dagger (G^\dagger G)_{IJ} U_J] \\ \times\left(\frac{1}{M_I}\gamma(\frac{1}{2}, t\frac{M_I^2}{M^0}) + \frac{1}{M_J}\gamma(\frac{1}{2}, t\frac{M_J^2}{M^0})\right) \tag{5.24}$$

We can now plug this in 5.13. For the asymmetry at the time of electroweak symmetry breaking any time t' after the critical time define in 5.21 we have

$$\Delta_{L,\alpha\beta} = \frac{49}{e2\times 96^2\pi^6}\frac{M^0}{\sqrt{2}}\int_0^{t'} dtT(t) \tag{5.25}$$
$$\times\,\Theta[\frac{3^{2/3}(M^0)^{1/3}}{2^{1/3}(\Delta M^2)^{2/3}} - t]\sum_{I,J}\left(\frac{1}{M_I}\gamma(\frac{1}{2},t\frac{M_I^2}{M^0}) + \frac{1}{M_J}\gamma(\frac{1}{2},t\frac{M_J^2}{M^0})\right)$$
$$\times\,\mathrm{diag}\left(G_{\alpha J}(G^\dagger G)_{JI}G^\dagger_{I\beta} - (G_{\alpha J}(G^\dagger G)_{JI}G^\dagger_{I\beta})^*\right)$$
$$= \frac{2\times 49}{e96^2\pi^6}\frac{(M^0)^{3/2}}{2}\int_0^{\frac{3^{2/3}(M^0)^{1/3}}{2^{1/3}(\Delta M^2)^{2/3}}} dt\,t^{-1/2}$$
$$\times\sum_{I>J}\left(\frac{1}{M_I}\gamma(\frac{1}{2},t\frac{M_I^2}{M^0}) + \frac{1}{M_J}\gamma(\frac{1}{2},t\frac{M_J^2}{M^0})\right)\mathrm{diag}\left(\mathrm{Im}[G_{\alpha J}(G^\dagger G)_{JI}G^\dagger_{I\beta}]\right)$$

Here we have used the fact that the $I = J$ terms do not contribute. If we now had $I = J$, the imaginary part of the matrix would disappear, as $(G_{\alpha I}(G^\dagger G)_{II}G^\dagger_{I\alpha})* = G^\dagger_{I\alpha}(G^\dagger G)_{II}G_{\alpha I} = G_{\alpha I}(G^\dagger G)_{II}G^\dagger_{I\alpha}$. We now need to calculate the integral over the t-dependent part

$$I = \int_0^{\frac{3^{2/3}(M^0)^{1/3}}{2^{1/3}(\Delta M^2)^{2/3}}} dt\,t^{-1/2}\gamma(\frac{1}{2},t\frac{M_I^2}{M^0}) \tag{5.26}$$

As $tM_I^2/M_0 \ll 1$ we can expand the gamma function to

$$\gamma(a,l) = \int_0^l dm\,m^{a-1}e^{-m} = \int_0^l dm\,m^{a-1}(1 - m + \frac{m^2}{2} + \dots) \tag{5.27}$$
$$\approx \int_0^l dm\,m^{a-1} = \frac{l^a}{a}$$

This gives us

$$I = \int_0^{\frac{3^{2/3}(M^0)^{1/3}}{2^{1/3}(\Delta M^2)^{2/3}}} dt\frac{2M_I}{\sqrt{M_0}} = \frac{3^{2/3}(M^0)^{1/3}}{2^{1/3}(\Delta M^2)^{2/3}}\frac{2M_I}{\sqrt{M_0}} \tag{5.28}$$

Substituting this back to 5.25

$$\Delta_{L,\alpha\beta} = \frac{2\times 49\times 3^{2/3}}{e96^2\times 2^{1/3}\pi^6}\sum_{I>J}\frac{(M^0)^{4/3}}{(\Delta M^2)^{2/3}}\left(\frac{2M_I}{M_I} + \frac{2M_J}{M_J}\right) \tag{5.29}$$
$$\times\,\mathrm{diag}\left(\mathrm{Im}[G_{\alpha J}(G^\dagger G)_{JI}G^\dagger_{I\beta}]\right)$$
$$= \frac{2\times 49\times 3^{2/3}}{e48^2\times 2^{1/3}\pi^6}\sum_{I>J}\frac{(M^0)^{4/3}}{\Delta(M^2)^{2/3}}\mathrm{diag}\left(\mathrm{Im}[G_{\alpha J}(G^\dagger G)_{JI}G^\dagger_{I\beta}]\right)$$

For generation α the asymmetry is

$$\Delta_{L,\alpha} = \frac{2 \times 49 \times 3^{2/3}}{e48^2 \times 2^{1/3}\pi^6} \sum_{I>J} \frac{(M^0)^{4/3}}{(\Delta M^2)^{2/3}} \mathrm{diag}\left(\mathrm{Im}[G_{\alpha J}(G^\dagger G)_{JI} G^\dagger_{I\alpha}]\right) \quad (5.30)$$

Note that here there is no summation over α. In fact, as $Tr[(GG^\dagger GG^\dagger)^*] = Tr[GG^\dagger GG^\dagger]$, the sum over generations would be zero. What this means is that at this point, net active lepton number is still conserved. There is summation over I and J, but they cannot be interpreted directly as matrix multiplication because of mass part ΔM^2.

The measure of the CP-violation here is $\delta^\alpha_{IJ} = \mathrm{Im}[G_{\alpha J}(G^\dagger G)_{JI} G^\dagger_{I\alpha}]$. Using the definition of G in 2.18 and taking into account that the mixing angles θ_{R12} and θ_{R13} are small, we can write

$$\begin{aligned} G^\dagger G &= P^\dagger_\beta K_R h_d^2 K_R^\dagger P_\beta \quad (5.31) \\ &= P^\dagger_\beta \begin{pmatrix} h_1^2 & 0 & 0 \\ 0 & c^2_{R23} h_2^2 + s^2_{R23} h_3^2 & c_{R23} s_{R23}(h_3^2 - h_2^2) \\ 0 & c_{R23} s_{R23}(h_3^2 - h_2^2) & c^2_{R23} h_3^2 + s^2_{R23} h_2^2 \end{pmatrix} P_\beta \end{aligned}$$

The complex matrices P_β annihilate with those of the surrounding G:s giving a real matrix. This means that the imaginary angles only come from the active particle side of the mixing matrix.

$$G_{\alpha I}(G^\dagger G)_{IJ} G^\dagger_{J\alpha} = K_{L,\alpha M} P_{\alpha,M} h_M K^\dagger_{R,MI} (K_R h_d^2 K_R^\dagger)_{IJ} K_{R,JN} h_N P^\dagger_{\alpha,N} K^\dagger_{L,N\alpha}$$

It is rather easy then to go through all the elements and take only those that have an imaginary part. Especially all diagonal elements ($I = J$) disappear as noted before. This leaves us only with $I = 3$ and $J = 2$.

$$\begin{aligned} \delta^e_{32} &= -[c_{R23} s_{R23}(h_3^2 - h_2^2)][h_2 h_3 c_{L13} s_{L12} s_{L13} \sin(\alpha_2 + \delta_L)] \quad (5.32) \\ \delta^\mu_{32} &= [c_{R23} s_{R23}(h_3^2 - h_2^2)] \\ &\quad \times [-h_2 h_3 c_{L12} c_{L13} c_{L23} s_{L23} \sin(\alpha_2) + h_2 h_3 c_{L13} s_{L12} s_{L13} s^2_{L23} \sin(\alpha_2 + \delta_L)] \\ \delta^\tau_{32} &= [c_{R23} s_{R23}(h_3^2 - h_2^2)] \\ &\quad \times [h_2 h_3 c_{L12} c_{L13} c_{L23} s_{L23} \sin(\alpha_2) + h_2 h_3 c_{L13} c^2_{L23} s_{L12} s_{L13} \sin(\alpha_2 + \delta_L)] \end{aligned}$$

And

$$\Delta_{L,\alpha} = \frac{2 \times 49 \times 3^{2/3}}{e48^2 \times 2^{1/3}\pi^6} \frac{(M^0)^{4/3}}{(\Delta M^2)^{2/3}_{32}} \delta^\alpha_{32}. \quad (5.33)$$

Clearly the angles δ_L and α_2 are important in the CP violation. As we noticed before, total active lepton number is not violated. This asymmetry is partly transferred to baryon number, but it is nevertheless far too small to be the real cause of the asymmetry we see. This is not, of course, the end of the story, these asymmetries generate an even greater active lepton asymmetry. This happens after the initial asymmetry is generated and actually causes the baryon asymmetry.

To find the asymmetry, we can consider equation 5.10. Because of the lepton asymmetry, the third term becomes nonzero and starts to generate sterile neutrinos. As total lepton number has to be conserved, the produced asymmetry has to be transferred back to active neutrinos. In the first order, we have now for sterile neutrino asymmetry $\Delta_N = \rho_{NN} - \rho_{\bar{N}\bar{N}}$

$$\begin{aligned}\frac{d\Delta_N}{dt} &= \frac{7T}{96\pi^3} U \Big[-U^\dagger(t) G^\dagger (\tilde{\rho}_{LL} - \tilde{\rho}^{eq}_{LL}) G U(t) \\ &+ U \dagger (t) G^\dagger (\tilde{\rho}_{\bar{L}\bar{L}} - \tilde{\rho}^{eq}_{\bar{L}\bar{L}}) G U(t) \Big] U^\dagger \\ &= -\frac{7T}{96\pi^3} G^\dagger(\Delta_L) G\end{aligned} \tag{5.34}$$

As the lepton asymmetry is constant after the time of its generation, this is easy to integrate. Again using

$$T = \sqrt{\frac{M^0}{2t}} \tag{5.35}$$

we have at the time of electroweak symmetry breaking, t_{EW}

$$\begin{aligned}\Delta_N^{\alpha\gamma} &= -\frac{7(M^0)^{1/2}}{96\pi^3\sqrt{2}} \int_0^{t_{\mathrm{EW}}} dt\, t^{-1/2} \sum_\beta G^\dagger_{\alpha\beta} (\Delta_L^\beta) G_{\beta\alpha} \\ &= -\frac{7(M^0)^{1/2}}{48\pi^3\sqrt{2}} t_{\mathrm{EW}}^{1/2} \sum_\beta G^\dagger_{\alpha\beta} (\Delta_L^\beta) G_{\beta\gamma} \\ &= -\frac{7^3 \times 3^{2/3}}{e48^3 \times 2^{1/3}\pi^9} \frac{(M^0)^{7/3}}{T_{\mathrm{EW}} (\Delta M^2)_{32}^{2/3}} \sum_\beta G^\dagger_{\alpha\beta} \delta_{32}^\beta G_{\beta\gamma}\end{aligned} \tag{5.36}$$

Since total lepton number is conserved, the traces of the asymmetries have to be same. Total active lepton asymmetry is then

$$\begin{aligned}\Delta_{L\,tot} = -Tr[\Delta_N] &= \frac{7^3 \times 3^{2/3}}{e48^3 \times 2^{1/3}\pi^9} \frac{(M^0)^{7/3}}{T_{\mathrm{EW}} (\Delta M^2)_{32}^{2/3}} \sum_{\alpha,\beta} G^\dagger_{\alpha\beta} \delta_{32}^\beta G_{\beta\alpha} \\ &= \frac{7^3 \times 3^{2/3}}{e48^3 \times 2^{1/3}\pi^9} \frac{(M^0)^{7/3}}{T_{\mathrm{EW}} (\Delta M^2)_{32}^{2/3}} \sum_{\alpha,\beta} \delta_{32}^\beta G_{\beta\alpha}^2\end{aligned} \tag{5.37}$$

This asymmetry is then converted into baryon number by the sphaleron processes. They freeze at $t = t_{\mathrm{EW}}$ and we calculated the ratio of baryon number to the total asymmetry in section 3.2. Now using 5.37 we find the baryon asymmetry

$$\Delta_B = \frac{28}{79}\Delta_{L\,tot} = \frac{7^4 \times 3^{2/3}}{e158 \times 24^3 \times 2^{1/3}\pi^9}\frac{(M^0)^{7/3}}{T_{\rm EW}(\Delta M^2)^{2/3}_{32}}\sum_{\alpha,\beta}\delta^{\beta}_{32}G^2_{\beta\alpha} \quad (5.38)$$

$$= 9.74446 \times 10^{31}\frac{\rm GeV^{4/3}}{(\Delta M^2)^{2/3}_{32}}\sum_{\alpha,\beta}\delta^{\beta}_{32}G^2_{\beta\alpha}$$

This suggests that we have quite degenerate masses for the two heavier right-handed neutrinos. This in turn indicates, that to have the expected values for the masses of left-handed neutrinos, we should have $h_3^2 \gg h_2^2 \gg h_1^2$. We now have

$$G^2_{\beta\alpha} = (K_{L,\beta 3}h_3K_{R,3\alpha})^2 \quad (5.39)$$
$$G^2_{12} = h_3^2s^2_{L13}s^2_{R23} \qquad G^2_{13} = h_3^2c^2_{R23}s^2_{L13}$$
$$G^2_{22} = h_3^2c^2_{L13}s^2_{L23}s^2_{R23} \qquad G^2_{23} = h_3^2c^2_{L13}c^2_{R23}s^2_{L23}$$
$$G^2_{32} = h_3^2c^2_{L13}c^2_{L23}s^2_{R23} \qquad G^2_{33} = h_3^2c^2_{L13}c^2_{L23}c^2_{R23}$$

Using these and 5.32 we have

$$\sum_{\alpha,\beta}\delta^{\beta}_{32}G^2_{\beta\alpha} = \quad (5.40)$$
$$- h_2h_3^5c_{R23}s_{R23}s^2_{L13}c_{L13}s_{L12}s_{L13}\sin(\alpha_2+\delta_L)$$
$$+ h_2h_3^5c_{R23}s_{R23}c^2_{L13}s^2_{L23}$$
$$\times[-c_{L12}c_{L13}c_{L23}s_{L23}\sin(\alpha_2) + c_{L13}s_{L12}s_{L13}s^2_{L23}\sin(\alpha_2+\delta_L)]$$
$$+ h_2h_3^5c_{R23}s_{R23}c^2_{L13}c^2_{L23}$$
$$\times[c_{L12}c_{L13}c_{L23}s_{L23}\sin(\alpha_2) + c_{L13}c^2_{L23}s_{L12}s_{L13}\sin(\alpha_2+\delta_L)]$$
$$= h_2h_3^5c_{R23}s_{R23}\{c^3_{L13}(c^2_{L23}-s^2_{L23})c_{L12}c_{L23}s_{L23}\sin(\alpha_2)$$
$$+ [c^2_{L13}(c^4_{L23}+s^4_{L23}) - s^2_{L13}]c_{L13}s_{L12}s_{L13}\sin(\alpha_2+\delta_L)\}$$
$$\equiv h_2h_3^5\delta_{\rm CP}$$

To get some more numerical values we can use as the rough values for the Yukawa couplings

$$h_2^2 = \frac{m_{sol}M_2}{v^2}, \qquad h_2^2 = \frac{m_{atm}M_3}{v^2}. \quad (5.41)$$

These would give neutrinos masses $m_{\nu_\mu} = m_{sol} \approx 8.9 \times 10^{-3}eV$ and $m_{\nu_\tau} = m_{atm} \approx 49 \times 10^{-3}$eV. We can now write the result for the baryon asymmetry

$$\Delta_B = 9.74446 \times 10^{31}{\rm GeV}^{4/3}\delta_{\rm CP}\frac{m_{sol}^{\frac{1}{2}}m_{atm}^{\frac{5}{2}}M_2^{\frac{1}{2}}M_3^{\frac{5}{2}}}{v^6(\Delta M^2)^{2/3}_{32}} \quad (5.42)$$

$$= 2.16235 \times 10^{-14}\delta_{\rm CP}\frac{(M_2^{\frac{1}{2}}M_3^{\frac{5}{2}})/{\rm GeV}^3}{(\Delta M^2)^{2/3}_{32}/{\rm GeV}^{4/3}}$$

5.2 Parameters of the asymmetry

There are now three undetermined parameters left in equation 5.42. These are the masses of the heavier sterile neutrinos M_2 and M_3, and CP violation parameter $\delta_{\rm CP}$. In order to compare our result to the measured values 3.3 we should normalize it to radiation density. ρ tells us the average occupation numbers density and its values are in the range $\rho \in [0, 1]$, much like the distribution functions. This means that

$$\Delta_B = \frac{N_B - N_{\bar{B}}}{N_B + N_{\bar{B}}} = \frac{n_B}{n_B^{eq}}, \tag{5.43}$$

where N_B is the amount of baryons and n_B is the baryon number density. For the equilibrium densities of baryon number and photons we have

$$\begin{aligned} n_B^{eq} &= 6 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{e^{p/T}+1} = \frac{9\zeta(3)T^3}{2\pi^2} \\ n_\gamma &= 2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{e^{p/T}-1} = \frac{2\zeta(3)T^3}{\pi^2} \\ \Rightarrow \eta &= \frac{n_B}{n_\gamma} = \frac{4}{9}\Delta_B \approx 9.61046 \times 10^{-15} \delta_{CP} \frac{(M_2^{\frac{1}{2}} M_3^{\frac{5}{2}})/\text{GeV}^3}{(\Delta M^2)_{32}^{2/3}/\text{GeV}^{4/3}} \end{aligned} \tag{5.44}$$

We should have

$$4 * 10^{-10} \geq \eta \geq 7 * 10^{-10}. \tag{5.45}$$

This is satisfied, for example, with $M_3 = M_2 = 10\text{GeV}$ and $\Delta M^2/M_3^2 = 2 \times 10^{-5}$, if $\delta_{\rm CP} = 1$. It is clear that the theory allows a range of parameters where the observed baryon asymmetry is generated. The two heavier sterile neutrinos have to be very degenerate in masses and there are limitation to the mixing angles of neutrinos. The CP violating angles δ_L and α_2 are also important to the process.

To get a better picture of the allowed parameters, let us take $\eta = 5.5 \times 10^{-10}$ as a characteristic value for the baryon asymmetry. This gives us

$$\frac{(\Delta M^2)_{32}}{M_3^2} = 7.30419 \times 10^{-8} \left(\delta_{\rm CP} \frac{M_2^{1/2} M_3^{7/6}}{\text{GeV}^{5/3}} \right)^{3/2} \tag{5.46}$$

Since $\delta_{\rm CP}$ is at most of order 1, the Majorana masses are truly quite degenerate, and we can take $M_2 = M_3 = M$, giving

$$\frac{(\Delta M^2)_{32}}{M_3^2} = 7.30419 \times 10^{-8} \delta_{\rm CP}^{3/2} \left(\frac{M}{\text{GeV}} \right)^{5/2} \tag{5.47}$$

Our discussion is valid only when the asymmetry is generated before electroweak symmetry breaking, so the generation temperature has to be higher than the electroweak scale

$$\left(\Delta M_{32}^2 M^0\right)^{1/3} \gtrsim 100\text{GeV} \tag{5.48}$$

giving

$$\Delta M_{32}^2 \gtrsim 1.410^{-12}\text{GeV}^2 \tag{5.49}$$

This also translates into a condition for the Majorana masses

$$M \gtrsim \frac{2.3\text{MeV}}{\delta_{\text{CP}}^{1/3}} \tag{5.50}$$

If these conditions didn't hold, we would have to analyze carefully what happens during the symmetry breaking and when the sphalerons freeze out. It might still be possible to generate the required asymmetry, but the phase transition would affect the result considerably. Even at the given range, strong fine tuning is required.

We have performed and order of magnitude analysis on the production of the baryon asymmetry in the νMSM. It clearly shows that there is a range of allowed parameters in which it is possible to explain both the baryon asymmetry and the existence of dark matter and, of course, the neutrino oscillations. The range is quite limited in terms of the Majorana masses and the corresponding Yukawa couplings are also somewhat limited. Nevertheless, the νMSM is clearly the most minimal extension to the standard model for explaining these cosmological problems. Furthermore, because no new mass scale is introduced, Inflation can be introduced in the model with a rather light scalar particle, inflaton [8]. The Majorana masses, as well as the Higgs mass, can then be explained trough couplings to the inflaton. These models share most of the advantages of the standard model, namely renormalizability and accordance with a lot of experimental data, but also its weaknesses, fine tuning problems such as the hierarchy problem and the flavour problem.

Chapter 6

Possible experiments

While the νMSM is theoretically tempting for many reasons, mainly its simplicity, what we want to know is if it really describes our world. It has to be experimentally testable. Though the limited parameter space poses theoretical problems, it is good news on the experimental side, as is the smallness of the masses of the new particles. Already existing experiments might cover the lower end of the mass spectrum and it might be at least possible to reach some of the energies of the higher end with current equipment. The problem with the experiments of the νMSM is the weakness of the interactions of sterile neutrinos. They only interact with the active neutrinos and all interactions are suppressed by the factor of m/M. This makes laboratory experiments extremely difficult. An overview of possible experiments of the νMSM is given in [26] and more detailed calculations can be found in [27].

There are two natural approaches to experimentally disproving the νMSM. The first would be to actually find the exotic particles responsible of GUT-baryogenesis or leptogenesis. The νMSM is based on the idea that no new physics except for the sterile neutrinos is required to explain cosmological phenomenology. Finding any new physics would of course disprove this idea. Nevertheless, the proposed mechanism for baryogenesis might not be disproved. If the found mechanism accounted for all of the baryon asymmetry, there would be no room for the asymmetry produced by oscillating sterile neutrinos. Even if they didn't, the theory would of course have to change radically include them and the allowed parameter range would change. The sterile neutrinos would, of course, still be required by the neutrino oscillations, but the scenario with small Majorana masses would be less plausible.

An anomalously large amount of positrons has been detected in cosmic ray experiments recently ([29], [30], [31]). These are believed to be produced in decays of dark matter particles [32]. The dark matter neutrino of the νMSM is far too light to produce positrons in its decays. Unless another source for the positrons is found, this sheds serious doubt on the theory. To identify the possible sources within the νMSM, a far more de-

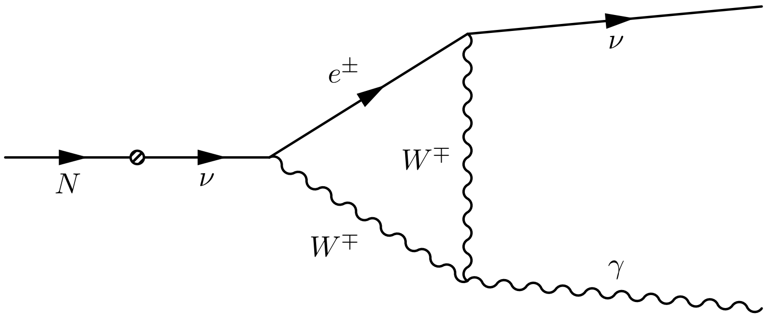


Figure 6.1: The radiative decay channel of the dark matter neutrino.

tailed discussion of the theory would be required, especially at temperatures below the electroweak symmetry breaking and around the temperatures of nucleosynthesis.

Of the sterile neutrinos, the dark matter particle N_1 is the lightest, and therefore most easily observed. It should also be the abundant in the Universe, making it the most promising one to be found. Its main decay channel $N\nu \to Z \to \nu\nu$ produces only a small neutrino flux that can not be observed. It has, however, a secondary radiative decay channel (figure 6.1). This produces a line at the energy $E = M_1/2$. The width of the line is only caused by the doppler effect due to the movement of neutrinos and it is very narrow. It could be observed in X-ray measurements.

No such line has yet been observed. This doesn't mean that it isn't there, but it isn't strong enough to be measured with current equipment. This constrains the mass ratio m_1/M_1 from up, but these constraints are small when the Majorana mass is of order 1keV. To actually test the theory, one would need better resolution, of the natural line width $\Delta E/E \sim 10^{-3}$, and have to scan the whole range of possible masses from $E \sim 100$eV to $E \sim 10$keV. Unfortunately, no new experiments that could significantly improve the constraints are planned.

X-ray observations still provide the best constraints on the dark matter neutrino mass and are probably the easiest way to study the validity νMSM. Were the decay line found, however, a possible laboratory experiment would be useful in confirming it. The small rate of processes involving the sterile neutrinos makes all laboratory experiments difficult. One might hope to create sterile neutrinos in colliders, or actually one should already have created them, as they have small mass. Detecting them, however, would be nearly impossible as all such processes would be suppressed by m_1^4/M_1^4. If neutrinos were generated in stars or supernovas, the ratio of sterile to active neutrinos would be boosted, as sterile neutrinos escape the plasma more easily. Nevertheless the rate of collisions of sterile neutrinos to those of active neutrinos would be very small. The only way to distinguish sterile neutrinos from active ones would be to study the kinematics of the collision and that would be nearly impossible due to the background noise.

One could still hope to create sterile neutrinos in decays of more massive, neutral particles. Normal decay channels are either too much suppressed or the energy lines of active and sterile decays are too close to each other to distinguished. A particularly tempting possibility would be the decay of a massive scalar boson S. It could not decay directly to active neutrinos as the process $S \to \nu\bar{\nu}$ is forbidden by conservation of chirality and $S \to \nu\nu$ by conservation of lepton number. With sterile neutrinos the process $S \to \nu N_1$ is possible. Observing a decay of the boson to unobservable neutral particles would thus prove the existence of sterile neutrinos and pose an upper limit to its mass. For more information one would need to be able to study the active neutrino generated. Unfortunately, the rates for such decays are however far too small to be studied seriously.

Another possibility is to use beta decays to find the sterile neutrinos. This could be done by full reconstruction of the decay event, measuring all kinematic information of the initial isotope, the produced ion and the electron. Finding just one certain anomalous decay would prove the existence of sterile neutrinos, though more statistics would be required to actually deduce any information about it. Such measurements have already been made, first at the time the active neutrino was discovered to test the theory of beta decay. More recently it has been used to hunt for heavier sterile neutrinos in the mass range $0.7 - 3.5$MeV [28].

One could do such measurement using, for example, the decay $^3H \to {}^3He + e + \bar{\nu}_e$. A possible setup was proposed in [26]. It is possible to measure the kinematics accurately enough. The main problem is again the small rate of the decays, which makes it hard to get enough statistics to compete with the X-ray measurements. This could be improved by using isotopes with greater decay energies and higher rates. Another problem is that at high energies extra photons are created in the decay process. These can be dealt with either by getting more statistics of by registering them with high efficiency.

All these experiments were designed to find the lightest of the sterile neutrinos. The two heavier sterile neutrinos could also be observed in particle accelerators. This requires the study of the branching ratios of their decay modes, which was done in [27] for different possible mass scales. There are basically three approaches. One is to study the kinematics of decays, searching for signs of productions of sterile neutrinos. Another is to search for decays of the heavy neutrinos. These would look like particles appearing out of nothing. One could also search for events in which the sterile neutrinos are created and decay in the same detector.

The heavier sterile neutrino masses are probably not at the range $M < m_\pi \approx 140$MeV, as this would spoil the predictions of nucleosynthesis and is already disfavored by experimental data. The range $m_\pi < M < m_{\mathrm{K}} \approx 500$MeV is allowed. At this energy range suitable experiments have already been done, but the statistics haven't been considered from this point of view. These could improve the boundaries on the masses. New experiments could completely exclude the existence of sterile neutrinos at this range.

The range $m_{\mathrm{K}} < M < 1\mathrm{GeV}$ could be studied trough the kinematics of decays of charmed mesons and the τ lepton. Above this, at $1\mathrm{GeV} < M < m_{\mathrm{D}} \approx 1.8\mathrm{GeV}$, it is unlikely that the necessary statistics could be accumulated. This range could best be studied trough the decays of the sterile neutrinos. These could be produced in hadronic colliders such as the LHC and decay in a detector at some distance of the collision point. Above the D-meson mass below the threshold for B-mesons ($m_B \approx 5\mathrm{GeV}$) it would be very hard to get meaningful results with any existing or planned colliders. For example for the CNGS at CERN, this would require either the increase of the intensity of the beam by two orders of magnitude or the statistics of more than 10^{10} B-mesons.

Actually scanning the possible parameters of the νMSM seems quite challenging. Though the particles are light, they interact very weakly and one would need to gather a lot of statistics in order to uncover them. The theory could however be falsified by very different, indirect observations. The discovery of the weakly interacting massive particles proposed to work as dark matter, or for that matter, any massive particle other than the Higgs boson, would falsify the model. Finding the active neutrinos to be degenerate in mass would also disprove it.

Chapter 7

Conclusions

The main motivation for introducing the right-handed neutrinos was explaining the neutrino oscillations in the simplest way, assuming that neutrinos have mass. The νMSM lagrangian was taken to be the most general one including all possible gauge invariant terms for the new particles. The only terms we omitted were the Majorana mass of the active neutrinos, as this would have to be very small according to experimental data.

Dark matter can then be explained by the nonzero density of the lightest sterile neutrino. Its lifetime is too long for it to have thermalized. This only requires that the dark matter neutrino is light enough, M_1 is at the keV scale. This also means that the Yukawa coupling determining the mass of the corresponding active neutrino has to be small enough. With these parameters, trough a process somewhat like leptogenesis, the observed amount of baryon asymmetry can be generated. The order of magnitude analysis shows that the required parameter space is limited, the two heavier sterile neutrinos have to be very degenerate in mass. The smaller the CP violating angles in the Yukawa couplings are, the more degenerate the masses have to be. This is a strong fine-tuning, but could also indicate the existence of a slightly broken symmetry.

Though the limited parameter range is a theoretical problem, it just might make experimental probing of the theory possible. At least the search for the dark matter sterile neutrino would be possible with current or planned equipment. Its radiative decay channel would probably be the easiest way of observing it. Even though finding the dark matter neutrino would not automatically prove the theory, it would strengthen it considerably. It would also be possible to scan some of the range of the heavier neutrino masses, but unfortunately not all of it, with current or planned equipment. The theory could also be disproved by finding other particles that explain the phenomenology its based on. The find of an excess of positrons, possibly produced by dark matter decays, already raises questions.

The νMSM is clearly the most minimal possible theory for explaining three observed physical phenomena: neutrino oscillations, dark matter and the baryon asymmetry. It introduces only the particles required by the neutrino masses. This simplicity is what makes it so tempting as a theory. Nevertheless, it shares with the standard model not only its successes in many experiments and renormalizability, but also its flaws, the fine tuning problems such as the hierarchy problem and the flavour problem. It also adds its own fine tuning problem, the great degeneracy of the neutrino masses. It is still a phenomenologically very successful theory worth careful experimental study.

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